

Progressive design methodology for complex engineering systems based on multiobjective genetic algorithms and linguistic decision making

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Abstract This work focuses on a design methodology that aids in design and development of complex engineering systems. This design methodology consists of simulation, optimization and decision making. Within this work a framework is presented in which modelling, multi-objective optimization and multi criteria decision making techniques are used to design an engineering system. Due to the complexity of the designed system a three-step design process is suggested. In the first step multi-objective optimization using genetic algorithm is used. In the second step a multi attribute decision making process based on linguistic variables is suggested in order to facilitate the designer to express the preferences. In the last step the fine tuning of selected few variants are performed. This methodology is named as progressive design methodology. The method is applied as a case study to design a permanent magnet brushless DC motor drive and the results are compared with experimental values.

Keywords BLDC motors · PDM · Voltage source inverter · Genetic algorithms · Linguistic variables · Multi attribute decision making · Multi-objective optimization (MOOP)

1 Introduction

The design of complex engineering systems involves many objectives and constraints and requires application of knowledge from several disciplines (multidisciplinary) of

engineering (Balling and Sobieszczanski 1996; Lewis and Mistree 1998; Sobieszczanski-Sobieski and Haftka 1997). The multidisciplinary nature of complex systems design presents challenges associated with modelling, simulation, computation time and integration of models from different disciplines. In order to simplify the design problems, assumptions based on the designer's understanding of the system are introduced. The ability and the experience of the designer usually lead to good but not necessarily an optimum design. Hence there is a need to introduce formal mathematical optimisation techniques, in design methodologies, to offer an organised and structured way to tackle design problems.

A review of different methods for design and optimisation of complex systems is given in (Tappeta et al. 1998; Sobieszczanski et al. 1998, 2000, 2002; Alexandrov and Lewis 2002). The increase in complexity of systems, as well as the number of design parameters needed to be co-ordinated with each other in an optimal way, have led to the necessity of using mathematical modelling of systems and application of optimisation techniques. In this situation the designer focuses on working out an adequate mathematical model and the analysis of the results obtained while the optimisation algorithms choose the optimal parameters for the system being designed. Marczyk (2000) presented stochastic simulation using the Monte Carlo technique as an alternative to traditional optimisation. In recent years probabilistic design analysis and optimisation methods have also been developed (Tong 2000; Koch et al. 2000; Egorov et al. 2002) to account for uncertainty and randomness through stochastic simulation and probabilistic analysis. Much work has been proposed to achieve high-fidelity design optimisation at reduced computational cost. Booker et al. (1999) developed a direct search Surrogate Based Optimisation (SBO) framework that converges to an

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objective function subject only to bounds on the design variables and it does not require derivative evaluation. Audet et al. (2000) extended that framework to handle general non-linear constraints using a filter for step acceptance (Audet and Dennis 2004).

The primary shortcoming of many existing design methodologies is that they tend to be hard coded, that is they are discipline or problem specific and have limited capabilities when it comes to incorporation of new technologies. There appears to be a need for a new methodology that can exploit different tools, strategies and techniques which strive to simplify the design cycle of engineering systems. The other drawback of the existing methodologies is that the designer needs extensive knowledge of the process itself. In order to overcome these problems a new design methodology, progressive design methodology (PDM), has been proposed. In the following sections the details of PDM are laid down. In the next section the various steps of PDM are explained. In Sect. 3 the first step of PDM, viz. the Synthesis Phase is described. Section 4 deals with the second step of PDM, the Intermediate Analysis. An explanation of the third step of PDM, Final Analysis is given in Sect. 5. In Sect. 6 the application of the Synthesis Phase of PDM to design of a BLDC motor drive is presented. The application of Intermediate Analysis Phase to PDM to BLDC motor drive is given in Sects. 7 and 8 deals with the application Final Analysis Phase to BLDC motor drive design. Finally the conclusions are drawn in Sect. 9.

2 Progressive design methodology

A design method is a scheme for organising reasoning steps and domain knowledge to construct a solution (Dasgupta 1989). Design methodologies are concerned with the question of how to design whereas the design process is concerned with the question of what to design. A good design methodology has following characteristics (Shakeri 1998):

- Takes less time and causes fewer failures,
- produces better design,
- works for a wide range of design requirements,
- integrates different disciplines,
- consumes less resources: time, money, expertise,
- requires less information.

An ideal condition in the design of an engineering system will be if all the objectives and constraints can be expressed by a simple model. However in practical design problems this is seldom the case due to the complexity of the system. Hence a trade-off has to be made between the complexity of the model and time to compute the model.

A complex model will enable us to represent all the objectives and constraints of the system but will be computationally intensive. On the other hand a simple model will be computationally inexpensive but will limit the scope of objectives and constraints that can be expressed. In order to overcome this problem PDM consists of three main phases:

- Synthesis phase of PDM,
- intermediate analysis phase of PDM,
- final design phase of PDM.

Since in the first step (*synthesis phase*) of PDM the detailed knowledge is unavailable hence the optimisation process is exhaustive. If complex models are used in this phase then the computational burden will be overwhelming. In order to facilitate the initial optimisation process only those objectives and constraints are considered that can be expressed by simple mathematical models of the system. In the *synthesis phase* a set of feasible solutions (Pareto Optimal Solutions) is obtained. The Fig. 1 illustrates a set of Pareto Optimal Solutions for a problem where two objectives (f_1 and f_2) are simultaneously minimised. The set of feasible solutions is obtained by using *multi objective optimisation*. Hence the engineering design problem is a *multi objective optimisation problem (MOOP)*. The primary purpose of the *synthesis phase* is to develop simple models of the system and translate the problem as to a multi-objective optimisation problem. The details of the *synthesis phase* are explained in Sect. 3.

The most important task in engineering design problems, besides developing suitable mathematical models, is to generate various design alternatives and then to make preliminary decision to select a design or a set of designs that meets a set of criterion. Hence the engineering design problem is also a *multicriteria decision making (MCDM)* problem as well. In the conceptual stages of design, the

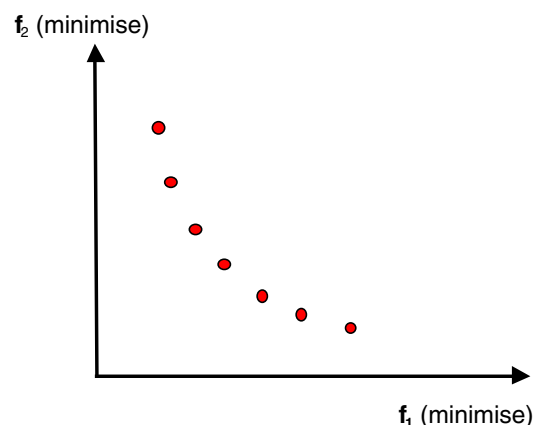


Fig. 1 A set of Pareto optimal solutions for an optimisation problem with two objectives

design engineer faces the greatest uncertainty in the product attributes and requirements (e.g. dimensions, features, materials and performance). The evolution of design is greatly affected by decisions made during the conceptual stage and these decisions have a considerable impact on overall cost.

In the *intermediate analysis phase multicriteria decision making* process is carried out. This step is a screening process where the set of solutions obtained from the *synthesis phase* is subjected to the process of screening. In order to achieve the screening additional constraints are taken into consideration. The constraints considered here are those that cannot be expressed explicitly in mathematical terms. The details of the *intermediate analysis phase* are given in Sect. 4.

In the *final design phase* detail model of the system is developed. After having executed the *synthesis phase* a better understanding of the system is obtained and it is possible to develop a detail model of the system. In this phase all the objectives and constraints that could not be considered in the *synthesis phase* are taken into consideration. In this phase exhaustive optimisation is not carried out, rather fine tuning of the variables is performed in order to satisfy all the objectives and constraints. The outline of the *final design phase* are given in Sect. 5.

3 Synthesis phase of PDM

In the *synthesis phase* the requirements of the system to be designed are identified. Based on these requirements system boundaries are defined and performance criterion/criteria are determined. The next step is to determine the independent design variables that will be changed during the optimisation process. The various steps involved in the *synthesis phase* are:

1. System requirements analysis,
2. definition of system boundaries,
3. determination of performance criterion/criteria,
4. selection of variables and sensitivity analysis,
5. development of system model,
6. deciding on the optimisation strategy.

The implementation of the above steps is shown in Fig. 2. From Fig. 2 it can be seen that the six steps involved in the *synthesis phase* are not executed in purely sequential manner. After the sensitivity analysis has been done and a set of independent design variables (IDV) has been identified, the designer has to decide if the set of IDV obtained is appropriate to proceed with the modelling process. The decision about the appropriateness of the set of IDV can be made based on previous experience or discussions with other experts. If the set of IDV is not

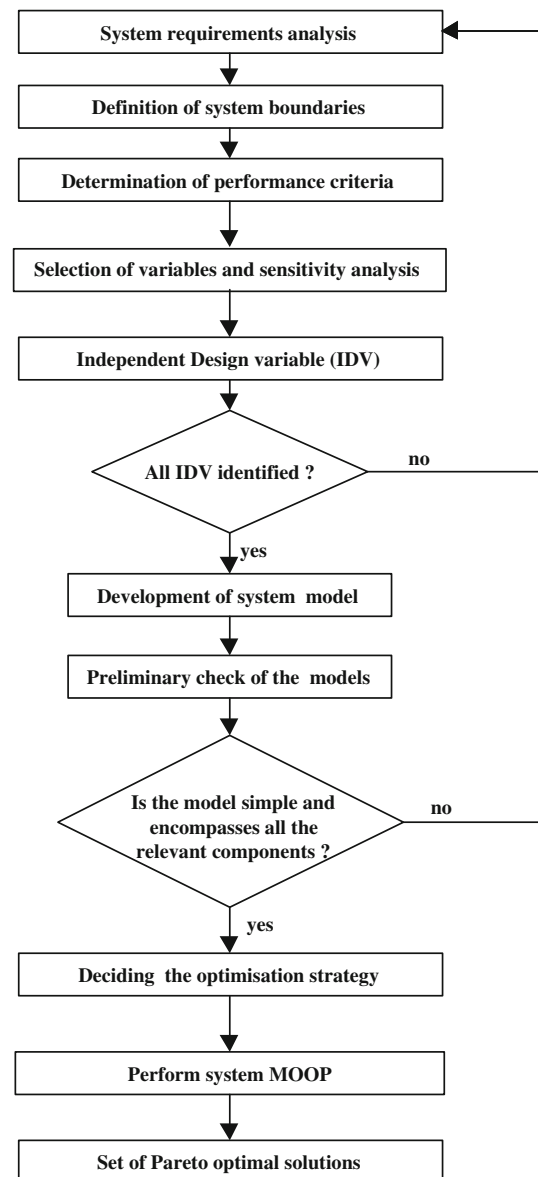
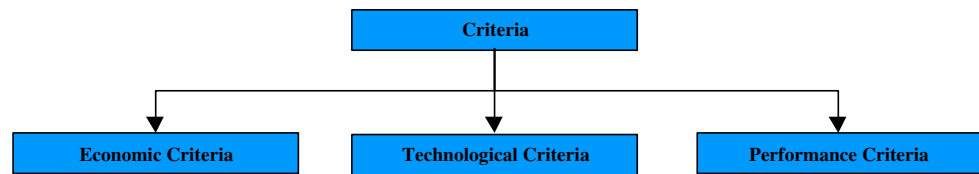


Fig. 2 Steps in the synthesis phase of progressive design methodology (PDM)

sufficient then it is prudent to go back to system requirement analysis and perform the loop again. This loop can be repeated until a satisfactory set of IDV is identified. Similarly after the model of the system to be designed (target system) is developed, it is important to check if the model includes the system boundaries and the set of IDV. In reality the selection of variables and the development of the model have to be done iteratively since both depend on each other. The choice of variables has influence on modelling and the modelling process itself will influence of the variables needed. The details of each of the above steps are given in the following subsections.

Fig. 3 Classification of criterion



3.1 System requirements analysis

The requirements of the system to be designed are analysed in this phase. The purpose of system requirement analysis is to develop a clear and detailed understanding of the needs that the system has to full fill. Hence this phase can be a challenging task since the requirements form the basis for all subsequent steps in the design process. The quality of the final product is highly dependent on the effectiveness of the requirement identification. The primary goal of this phase is to develop a detailed functional specification defining the full set of system capabilities to be implemented.

3.2 Definition of system boundaries

Before attempting to optimise a system, the boundaries of the system to be designed should be identified and clearly defined. The definition of the clear system boundaries helps in the process of approximating the real system (Chong and Zak 2001). Since an engineering system consists of many subsystems it may be necessary to expand the system boundaries to include those subsystems that have a strong influence on the operation of the system that is to be designed. As the boundaries of the system increases, i.e. more the number of subsystems to be included, the complexity of the model increases. Hence it is prudent to decompose the complex system into smaller subsystems that can be dealt with individually. However care must be exercised while decomposing the system as too much decomposition may result in misleading simplifications of the reality. For example a brushless direct current (BLDC) motor drive system consists of three major subsystems viz.

- The BLDC motor,
- voltage source inverter (VSI),
- feedback control.

Usually a BLDC motor is designed for a rated load, i.e. the motor is required to deliver a specified amount of torque at specified speed for continuous operation at a specified input voltage. During design process the motor is the primary system under design. However, optimised design of the motor based only on the magnetic circuit may result in misleading results. It is possible that this optimised motor has a high electrical time constant and the VSI is not able to provide sufficient current resulting in lower

torque at rated speed and given input voltage. Hence, for the successful design of the BLDC motor it is important to include the VSI in the system, i.e. the boundary of the system is expanded. Of course, it is a different matter that the model of the system that includes the BLDC motor and the VSI is more complicated but nevertheless is closer to the reality.

3.3 Determination of performance criterion/criteria

Once the proper boundaries of the system have been defined, performance criterion/criteria are determined. The criterion/criteria form the basis on which the performance of the system is evaluated so that the best design can be identified. In engineering design problems different types of criteria can be classified as depicted in Fig. 3 (Chong and Zak 2001):

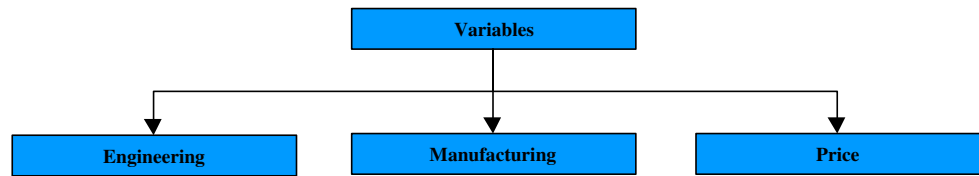
- Economic criterion/criteria: In engineering system design problems the economic criterion involves total capital cost, annual cost, annual net profit, return on investment, cost-benefit ration or net present worth.
- Technological criterion/criteria: The technological criterion involves production time, production rate, and manufacturability.
- Performance criterion/criteria: Performance criterion is directly related to the performance of the engineering system such as torque, losses, speed, mass, etc.

In the synthesis phase of PDM the *Performance criterion/criteria* are taken into consideration because they can be expressed explicitly in the mathematical model of the system. The economic and technological criteria are suitable for *Intermediate analysis* and *Final design phases* of PDM because by then detailed knowledge about the engineering systems performance and dimensions are available.

3.4 Selection of variables and sensitivity analysis

The next step is selection of variables that are adequate to characterise the possible candidate design. The design variables can be broadly classified as, Fig. 4:

- *Engineering variables*: The engineering variables are specific to the system being designed. These are variables with which the designer deals.
- *Manufacturing variables*: These variables are specific to the manufacturing domain.

Fig. 4 Classification of variables

- *Price variables*: This variable is the price of the product or the system being designed.

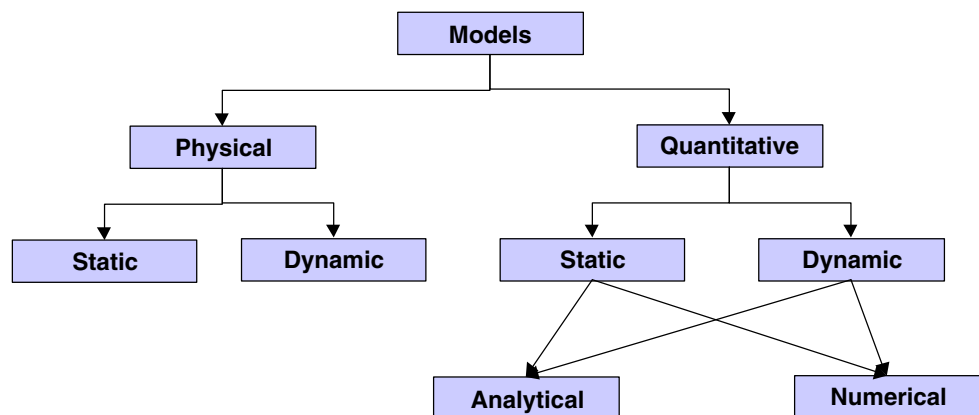
In the *synthesis* phase of PDM *engineering* variables are considered. There are two factors to be taken into account while selecting the engineering variables. First it is important to include all the important variables that influence the operation of the system or affect the design. Second, it is important to consider the level of detail at which the model of the system is developed. While it is important to treat all the key *engineering* variables, it is equally important not to obscure the problem by the inclusion of a large number of finer details of secondary importance (Chong and Zak 2001). In order to select the proper set of variables, sensitivity analysis is performed. For sensitivity analysis all the *engineering* variables are considered and its influence on the objective parameters is considered. The sensitivity analysis enables to discard the *engineering* variables that have least influence on the objectives.

3.4.1 Development of system model

A model is any incomplete representation of reality, an abstraction but could be close to reality. The purpose in developing a model is to answer a question or a set of questions. If the questions that the model has to answer, about the system under investigation, are specific then it is easier to develop a suitable and useful model. The models that have to answer a wide range of questions or generic

questions are most difficult to develop. The most effective process for developing a model is to begin by defining the questions that the model should be able to answer. Broadly models can be classified into following categories (Buede 1999), Fig. 5:

- *Physical models*: These models are full-scale mock-up, sub-scale mock-up or electronic mock up.
- *Quantitative models*: These models give numerical answers. These models can be either analytical, simulation or judgmental. These models can be dynamic or static. An analytical model is based on system of equations that can be solved to produce a set of closed form solutions. However finding exact solutions of analytical equations is not always feasible. Simulation models are used in situations where analytical models are difficult to develop or are not realistic. The main advantage of analytical models is that they are faster than numerical models and hence are suited for MOOP. The major aspect of analytical model is that certain approximations are required to develop analytical models. However in certain cases where approximations cannot be made and a very deep insight of the system are required then numerical simulation methods such as Finite element method (FEM), Computational fluid dynamics (CFD), etc. have to be adopted. The main drawback of numerical models is that they are computationally intensive and are not suitable for exhaustive optimisation process.

Fig. 5 Classification of models

3.5 Deciding on optimising strategy

Multi-objective optimisation results in a set of Pareto optimal solutions specifying the design variables and their objective tradeoffs. These solutions can be analysed to determine if there exist some common principles between the design variables and the objectives (Deb and Srinivasan 2005). If a relation between the design variables and objectives exist they will be of great value to the system designers. This information will provide knowledge of how to design the system for a new application without resorting to solving a completely new optimisation problem again.

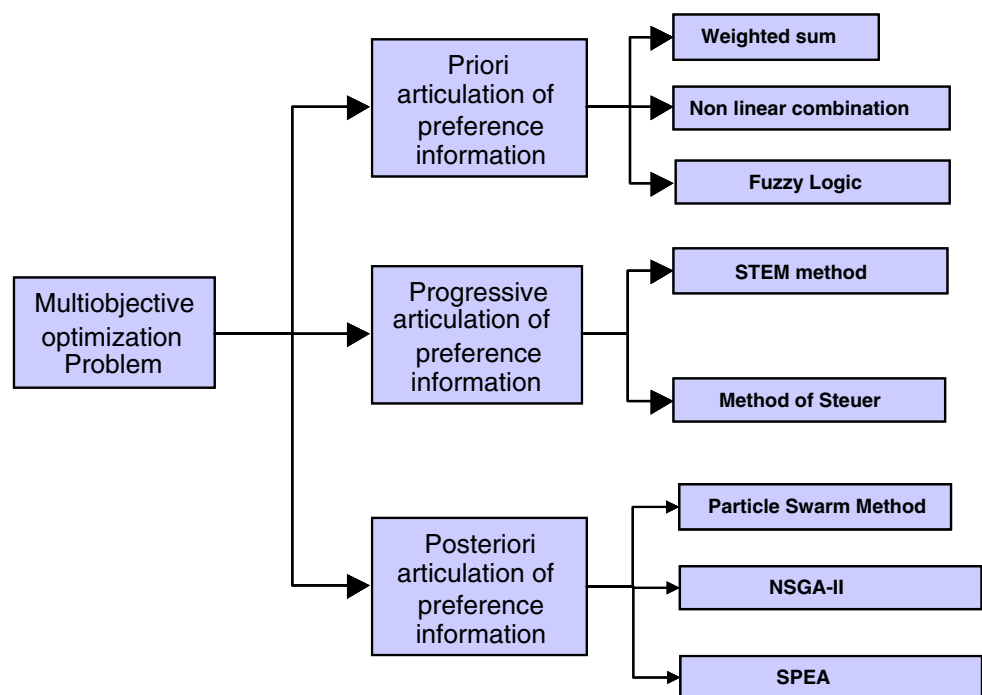
The principles of multi-objective optimisation are different from that of a single objective optimisation. When faced with only a single objective an optimal solution is one that minimises the objective subject to the constraints. However, in a multi-objective optimisation problem (MOOP) there are more than one objective functions and each of them may have a different individual optimal solution. Hence, many solutions exist for such problems. The MOOP can be solved in four different ways depending on when the decision maker articulates his preference concerning the different objectives (Hwang and Yoon 1980). The classification of the strategies is as follows, Fig. 6:

- *Priori articulation of preference information*: In this method the DM gives his preference to the objectives before the actual optimisation is conducted. The objectives are aggregated into one single objective

function. Some of the optimisation techniques that fall under this category are *weighted-sum approach* (Steuer 1986; Das and Dennis 1997), *Non-Linear approaches* (Krus et al. 1995), *Utility theory* (Krus et al. 1995; Thurston 1991).

- *Progressive articulation of preference information*: In this method the DM indicates the preferences for the objectives as the search moves and the decision-maker learns more about the problem. In these methods the decision maker either changes the weights in a *weighted-sum approach* (Steuer and Choo 1983), or by progressively reducing the search space as in the *STEM method* of reference (Benayoun et al. 1971). The advantages of this method are that it is a learning process where the decision-maker gets a better understanding of the problem. Since the DM is actively involved in the search it is likely that the DM accepts the final solution. The main disadvantage of this method is that a great degree of effort is required from the DM during the entire search process. Moreover the solution depends on the preference of one DM and if the DM changes his/her preferences or if a new DM comes then the process has to restart.
- *Posteriori articulation of preference information*: In this method the search space is scanned first and Pareto optimal solutions are identified. This set of Pareto optimal solution is finally presented to DM. The main advantage of this method is that the solutions are independent of DM's preferences. The process of optimisation is performed only once and Pareto optimal

Fig. 6 Classification of optimisation methods based on aggregation of information



set does not change as long as the problem description remains unchanged. The disadvantage of this method is that they need large number of computations to be performed and the DM is presented with too many solutions to choose from.

The principle goal of multi-objective optimisation algorithms is to find well spread set of Pareto optimal solutions. Each of the solutions in the Pareto optimal set corresponds to the optimum solution of a composite problem trading-off different objective among the objectives. Hence each solution is important with respect to some trade-off relation between the objectives. However in real situations only one solution is to be implemented. Therefore, the question arises about how to choose among the multiple solutions. The choice may not be difficult to answer in the presence of many trade-off solutions, but is difficult to answer in the absence of any trade-off information. If a designer knows the exact trade-off among objective functions there is no need to find multiple solutions (Pareto optimal solutions) and a priori articulation methods will be well suited. However, a designer is seldom certain about the exact trade-off relation among the objectives. In such circumstance it is better to find a set of Pareto optimal solutions first and then choose one solution from the set by using additional higher level information about the system being designed. With this in view in PDM posteriori based optimisation method is used. In principle any posteriori based multiobjective optimisation algorithm such as NSGA-II (Deb et al. 2000), SPEA 2 (Zitzler et al. 2001), etc. can be used in PDM. In this work the NBGA (Kumar et al. 2006) was used. Choosing a suitable solution from the Pareto optimal set forms the second phase of PDM and is described in the next section.

4 Intermediate analysis phase of PDM

Once the synthesis process is done and a set of Pareto optimal solutions is determined the next step involves analysis of the solutions. In the conceptual stages of design, the design engineer faces the greatest uncertainty in the product attributes and requirements (e.g. dimensions, features, materials, and performance). Because the evolution of the design is greatly affected by decisions made during the conceptual stage, these decisions have a considerable impact on overall cost. In the *intermediate analysis* phase the various alternatives obtained from the previous step (*synthesis phase*) are analysed and a small set of solutions are selected for deeper analysis. The most important tasks in engineering design, besides modelling

and simulation, are to generate various design alternatives and then to make preliminary decision to select a design or a set of designs that fulfils a set of criteria. Hence the engineering design decision problem is a multi criteria decision-making problem.

It is a general assumption that evaluation of a design on the basis of any individual criterion is a simple and straightforward process. However in practice, the determination of the individual criterion may require considerable engineering judgement (Scott and Antonsson 1999). An extensive literature survey on multi criteria decision making is given in the work of Bana de Costa (Costa and Vincke 1990). Carlsson and Fuller (1996) gave a survey of fuzzy multi criteria decision making methods with emphasis on fuzzy relations between interdependent criteria. A new elicitation method for assigning criteria importance based on linguistic variables is presented in (Ribeiro 1996). Roubens (1997) introduced a new pair wise preferred approach that permitted a homogeneous treatment of different kinds of criteria evaluations. A fuzzy model for design evaluation based on multiple criteria analysis in engineering systems is presented by Martinez and Liu (2006).

In the initial phase of development of an engineering system the details of a design are unknown and design description is still imprecise that the most important decisions are made (Whitney 1988). In this initial engineering design phase, the final values of the design variables are uncertain. Hence at this stage decision making using fuzzy linguistic variables is appropriate. After a decision is made and an alternative or set of alternatives is selected, detailed modelling of the system using standard tools (such as finite element Analysis, etc) serve to calculate the performance of the system and also help in reducing the uncertainty in the design variables.

In the initial stage of decision making the designers represent their preferences for different values of design variables using a set of fuzzy linguistic variables. Each value of design variable is assigned a preference between absolutely unacceptable and absolutely acceptable. The values of design variables have linguistic preference values. Hence the designer's judgement and experience are formally included in the preliminary design problem. The general problem is thus a Multi Criteria Decision-Making problem, where the designer is to choose the highest performing design configuration from the available set of design alternatives and each design is judged by several, even competing, performance criteria or variables.

A multi criteria decision-making problem (mcdm) is expressed as:

$$D = \begin{matrix} & c_1 & c_2 & \dots & c_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \end{matrix}$$

$$w = (w_1, w_2, \dots, w_n)$$

where A_i , $i = 1, \dots, m$ are the possible alternatives; c_j , $j = 1, \dots, n$ are the criteria with which alternative performances are measured and x_{ij} is the performance score of the alternative A_i with respect to attribute c_j and w_j are the relative importance of attributes.

The alternative performance rating x_{ij} can be crisp, fuzzy, and/or linguistic. The linguistic approach is an approximation technique in which the performance ratings are represented as linguistic variable (Zadeh 1975a, b, c). The classical MCDM problem consists of two phases:

- an aggregation phase of the performance values with respect to all the criteria for obtaining a collective performance value for alternatives,

- an exploitation phase of the collective performance value for obtaining a rank ordering, sorting or choice among the alternatives.

The various parts of intermediate analysis phase of PDM are:

1. Identification of new set of criteria,
2. linguistic term set,
3. semantic of linguistic term set,
4. aggregation operator for linguistic weighted information.

The flow chart of the above steps is shown in Fig. 7.

4.1 Identification of new set of criteria

In the synthesis stage the constraints imposed on the system are engineering constraints. The *engineering constraints* are specific to the system being designed and can be considered as criteria based on which decision making is done. Besides *engineering constraints* there are other *non-engineering constraints* such as manufacturing limitations. It may be possible that certain Pareto optimal solutions obtained in the *synthesis* stage may not be feasible from the manufacturing point of view or may be too expensive to manufacture. Hence, in order to determine these constraints a high level of information is to be collected from various experts.

4.1.1 Linguistic term set

After determining all the constraints, the next step is to determine the *linguistic term set*. This phase consists of establishing the linguistic expression domain used to provide the linguistic performance values for an alternative according to different criteria. The first step in the solution of a MCDM problem is selection of linguistic variable set.

There are two ways to choose the appropriate linguistic description of term set and their semantic (Bordogna and Passi 1993). In the first case by means of a context-free grammar, and the semantic of linguistic terms is represented by fuzzy numbers described by membership functions based on parameters and a semantic rule (Bordogna and Passi 1993; Bonissone 1986). In the second case the linguistic term set by means of an ordered structure of linguistic terms, and the semantic of linguistic terms is derived from their own ordered structure which may be either symmetrically/asymmetrically distributed on the $[0,1]$ scale. An example of a set of seven terms of ordered structured linguistic terms is as follows:

$$S = \{s_0 = \text{none}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{medium}, s_4 = \text{high}, s_5 = \text{very high}, s_6 = \text{perfect}\}$$

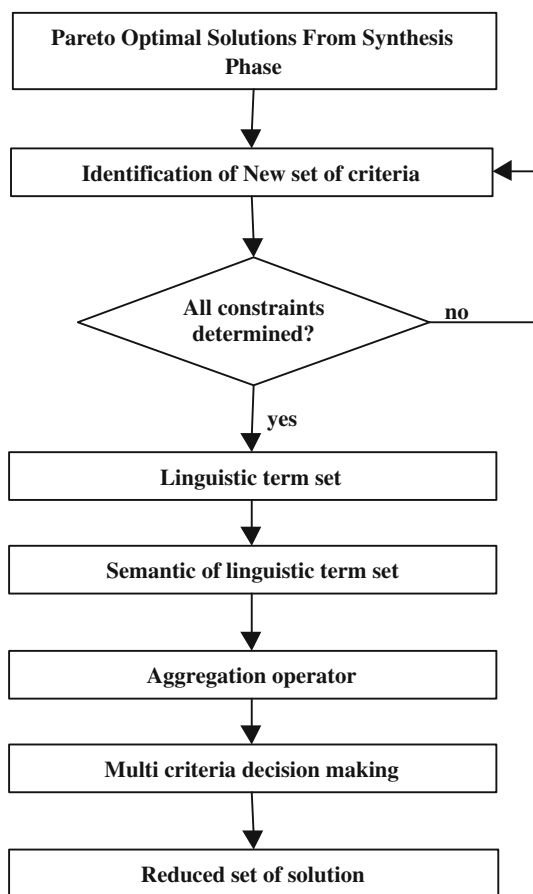


Fig. 7 Steps in the intermediate analysis phase of PDM

Fig. 8 Classification of semantic of linguistic term set

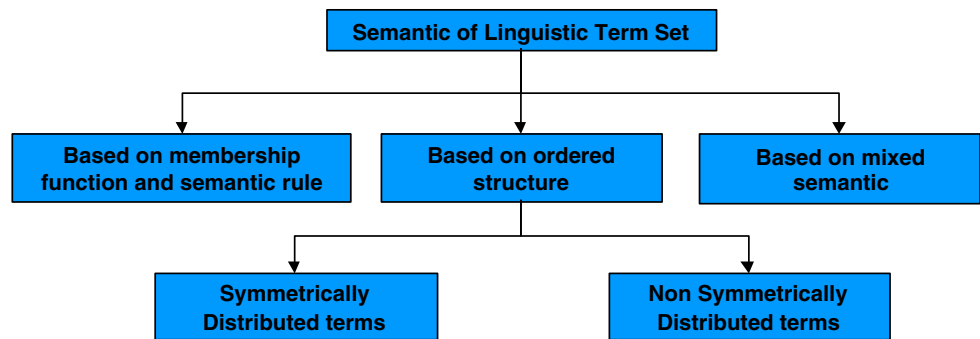
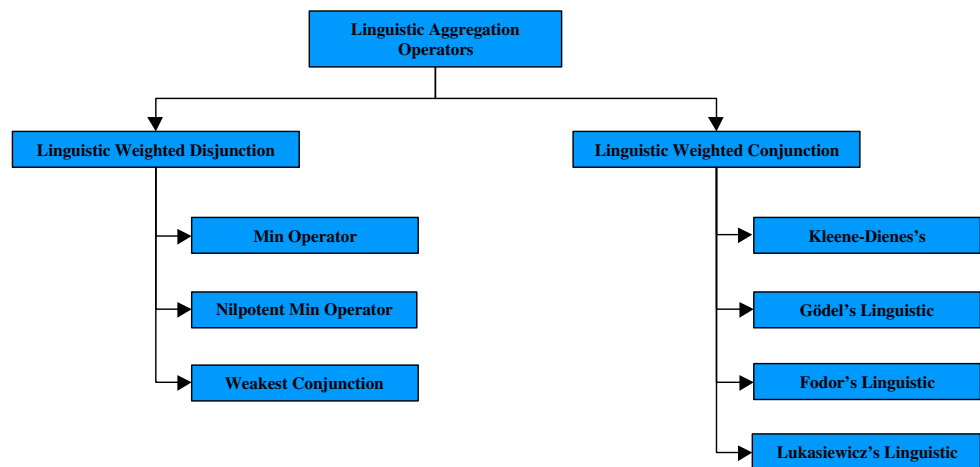


Fig. 9 Classification of aggregation operator for linguistic variables



4.1.2 The semantic of linguistic term set

The semantics of the linguistic term set can be broadly classified into three categories (Fig. 8), (a) Semantic based on membership functions and semantic rule (Bonissone and Decker 1986; Bordogna et al. 1997; Delgado et al. 1992), (b) Semantic based on the ordered structure of the linguistic term set (Bordogna and Passi 1993; Herrera and Herrera-Viedma 2000; Torra 1996; Herrera and Verdegay 1996) and (c) Mixed semantic (Herrera and Herrera-Viedma 2000; Herrera and Verdegay 1996).

4.1.3 Aggregation operator for linguistic weighted information

Aggregation of information is an important aspect for all kinds of knowledge based systems, from image processing to decision making. The purpose of aggregation process is to use different pieces of information to arrive at a conclusion or a decision. Conventional aggregation operators such as the weighted average are special cases of more general aggregation operators such as Choquet integrals (Gabrish et al. 1999). The conventional aggregation operators have been articulated with logical connectives arising from many-valued logic and interpreted as fuzzy set unions

or intersections (Dubois and Prade 2004). The latter have been generalised in the theory of triangular norms (Klement et al. 2000). Other aggregation operators that have been proposed are symmetric sums (Sivert 1979), null-norms (Calvo et al. 2001), uninorm (Yager and Fodor 1997), apart from other.

The aggregation operators can be grouped into the following broad classes (Dubois and Prade 2004):

- i. Operators generalising the notion of conjunction are basically the minimum and all those functions f bounded from above by the minimum operators.
- ii. Operators generalising the notion of disjunction are basically the maximum and all those functions f bounded from below by the maximum operations.
- iii. Averaging operators are all those functions lying between the maximum and minimum.

For linguistic weighted information the aggregation operators mentioned above have to be modified for linguistic variables and can be placed under two categories (Herrera and Herrera-Viedma 1997) Linguistic Weighted Disjunction (LWD) and Linguistic Weighted Conjunction (LWC). In Fig. 9 the detailed classification of the linguistic aggregation operators is shown. In the following subsections the mathematical formulation of LWD and LWC is

Table 1 The relation between numerical values and linguistic variables

| | N | VL | L | M | H | VH | OU |
|----|---------|---------|---------|---------|---------|---------|---------|
| C1 | 100–200 | 200–300 | 300–400 | 400–500 | 500–600 | 600–700 | 700–800 |
| C2 | 10–20 | 20–30 | 30–40 | 40–50 | 50–60 | 60–70 | 70–80 |
| C3 | 10–20 | 20–30 | 30–40 | 40–50 | 50–60 | 60–70 | 70–80 |
| C4 | 0.1–0.2 | 0.2–0.3 | 0.3–0.4 | 0.4–0.5 | 0.5–0.6 | 0.6–0.7 | 0.7–0.8 |
| C5 | 1–2 | 2–3 | 3–4 | 4–5 | 5–6 | 6–7 | 7–8 |

Table 2 Importance and score of alternative

| Criteria | C1 | C2 | C3 | C4 | C5 |
|------------------------|---------|---------|---------|-----------|----------|
| Importance weight (w) | VH | VH | M | L | VL |
| Score of alternative 1 | M (425) | L (34) | OU (77) | VH (0.65) | OU (7.6) |
| Score of alternative 2 | M (460) | OU (75) | VH (64) | VH (0.67) | H (5.6) |
| Score of alternative 3 | H (572) | M (47) | VH (64) | H (0.53) | OU (7.8) |
| Score of alternative 4 | OU (72) | M (45) | H (53) | VH (0.66) | H (5.8) |
| Score of alternative 5 | H (550) | M (46) | H (55) | OU (0.74) | VH (6.5) |

given. In order to illustrate each of the above mentioned linguistic aggregation operators the following example is considered (Carlsson and Fuller 1995):

Example: for each alternative an expert is required to provide his/her opinion in terms of elements from the following scale

$$S = \{OU(S_7), H(S_6), H(S_5), M(S_4), L(S_3), VL(S_2), N(S_1)\}$$

where OU stands for Outstanding, VH for Very High, H for High, M for Medium, L for Low, VL for Very Low, N for None. The expert provides the opinion on a set of five criteria $\{C_1, C_2, C_3, C_4, C_5\}$. An example of criteria as for electrical drive can be:

- C_1 Mass of the motor (Minimum mass is 100 gram and maximum mass is 800 gram)
- C_2 Cost of the electrical drive (Minimum cost is €10 gram and maximum cost is €80)
- C_3 Losses in the electrical drive (Minimum loss is 10 watts and maximum loss is 80 watts)
- C_4 Electrical time constant (Minimum loss is 0.1 milliseconds and maximum time constant is 0.8 milliseconds)
- C_5 Moment of inertia of the motor (Minimum moment of inertia is 1 and maximum moment of inertia is 8)

The performance of each alternative is also defined in terms of the scale $S = \{OU(S_7), VH(S_6), H(S_5), M(S_4), L(S_3), VL(S_2), N(S_1)\}$. The scale is evenly distributed and the scale for each alternative is given in Table 1 (Yager 1981).

The problem is to select a drive that has lowest losses, lowest cost, lowest mass, low electrical time constant and low moment of inertia. The motor is to be used in a hand held drill. For this application the mass of the motor and its cost are very important because a lighter motor with a low cost will be most preferred. Hence these two criteria are given Very High (VH) importance. For this application the efficiency of the motor is of moderate importance and is given a Medium (M) importance. The electrical time constant and moment of inertia of the rotor are important from the dynamic behaviour of the motor and are not very important for the application in hand held drill and are given low (L) and Very Low (VL) importance. The importance to each criterion is shown in Table 2. The performance of an alternative on all the criteria is also shown in Table 2; in brackets the numerical value is given.

The aggregation of the weighted information using Linguistic Weighted Conjunction (LWC) is defined as follows

$$f = LWC[(w_1, a_1), \dots, (w_m, a_m)]$$

where $LWC = \min_{i=1, \dots, m} \max(\text{Neg}(w_i), a_i)$ and m is the number of alternatives. An example of LWC is *Kleene-Dienes's Linguistic Implication Function* LI_1^- : (Herrera and Herrera-Viedma 1997)

$$LI_1^-(w, a) = \max(\text{Neg}(w), a)$$

Based on the example given in Tables 1, 2 the net performance of the first alternative based on LI_1^- is

$$f_1 = \min[LI_1^-(VH, M), LI_1^-(VH, L), LI_1^-(M, OU), \\ LI_1^-(L, VH), LI_1^-(VL, OU)] \\ = \min[M, L, OU, VH, OU] = L$$

The final score of the second alternative is

$$f_2 = \min[LI_1^-(VH, M), LI_1^-(VH, OU), LI_1^-(M, VH), \\ LI_1^-(L, VH), LI_1^-(VL, H)] \\ = \min[M, OU, VH, VH, VH] = M$$

The final score of the third alternative is

$$f_3 = \min[LI_1^-(VH, H), LI_1^-(VH, M), LI_1^-(M, VH), \\ LI_1^-(L, H), LI_1^-(VL, OU)] \\ = \min[H, M, VH, H, OU] = M$$

The final score of the fourth alternative is

$$f_4 = \min[LI_1^-(VH, OU), LI_1^-(VH, M), LI_1^-(M, H), \\ LI_1^-(L, VH), LI_1^-(VL, H)] \\ = \min[OU, M, H, VH, VH] = M$$

The final score of the fifth alternative is

$$f_5 = \min[LI_1^-(VH, M), LI_1^-(VH, M), LI_1^-(M, H), \\ LI_1^-(L, OU), LI_1^-(VL, H)] \\ = \min[M, M, H, OU, VH] = M$$

Hence on the basis of LI_1^+ the final score of all the alternatives is $[L, M, M, M, M]$.

The results of total score of all the five alternatives based on different aggregation operators is summarised below in Table 3.

From the above the following conclusions can be drawn:

- The choice of linguistic aggregation operator can influence the results of the intermediate analysis process.
- The linguistic weighted disjunction aggregation operators in general give an optimistic average value to alternatives. The Weakest linguistic disjunction gives the least optimistic value to the alternatives.

Table 3 The result of total score of all the alternatives using different aggregation operators

| Alternative \rightarrow | 1 | 2 | 3 | 4 | 5 |
|---------------------------|---|----|----|----|---|
| Min LD_1^+ | M | VH | H | VH | H |
| Nilpotent LD_2^+ | M | VH | H | VH | H |
| Weakest LD_3^+ | M | VH | VL | VH | L |
| Kleene-Dienes's LI_1^+ | L | M | M | M | M |
| Gödel's LI_2^+ | L | M | M | M | M |
| Fodor's LI_3^+ | L | M | M | M | M |
| Lukasiewicz's LI_4^+ | M | H | M | H | H |

- The linguistic weighted conjunction aggregation operators in general give a pessimistic average value to the alternatives.
- Out of all the conjunction operators the Lukasiewicz's implication operator gives the least pessimistic final score to all the alternatives.
- The disjunction aggregation operators are suitable if it is required to select a set of as many alternatives as possible. This situation can arise in the initial design phase when the designer wants to include as many alternatives as possible for further investigation.
- In the initial design process if the number of alternatives is large and there is limited capability, in terms of manpower and computing power, to investigate each alternative then linguistic weighted conjunction operators are preferred.

5 Final analysis phase of PDM

In the final analysis detailed simulation model of the target system is developed. After intermediate analysis the set of plausible solutions is greatly reduced and hence a detailed simulation for each solution is feasible. After setting up of the simulation model a new set of *Independent design variables* and *objectives* is identified. The steps involved in this stage are:

- Step1: Detailed simulation model of the target system is developed.
- Step2: *Independent design variables* and *objectives* are identified.
- Step3: Each solution in the reduced solution set is optimised for the new *objectives* and a set of solutions is obtained.
- Step4: Final decision is made.

In the next section the PDM is applied for design of a BLDC motor. The various aspects of PDM are used in the design of BLDC motor.

6 Synthesis phase of progressive design methodology for design of a BLDC motor drive

Since the emergence of new high field permanent magnet materials brushless DC motors (BLDC) drives have become increasingly attractive in a wide range of applications. They have smaller volume compared with equivalent wound field machines, operate at higher speed, dissipate heat better, require less maintenance, and are more efficient and reliable than conventional motors.

Many researchers have made efforts to improve motor performance in terms of efficiency, maximum torque, back EMF, power/ weight ratio, and minimum losses in iron, coils, friction, and windage. A scheme for optimisation of a three phase electric motor based on genetic algorithms (GA) was presented by Bianchi (1998). As a demonstration of this technique the authors took a surface mounted permanent magnet motor as an example and applied genetic algorithm to minimise the permanent magnet weight. Similarly an optimal design of Interior Permanent Magnet Synchronous Motor using genetic algorithms was performed by Sim et al. (1997). In this case the efficiency of the motor was taken as the objective function. In recent years research has been pursued in the area of multiobjective optimisation of PM motors. Multiobjective optimisation of PM motor using genetic algorithms was performed by Yamada et al. (1997). A surface mounted PM synchronous motor was taken for optimisation and ϵ -constraint method was used to obtain the solution. The objective functions that were considered for optimisation were motor weight and material cost. The authors used a two step method for optimisation. First a preliminary design was carried out in which the design is formulated as a constraint non-linear programming problem by using space harmonic analysis. Then the motor configuration was optimised using a procedure that combined the finite element method (FEM) with the optimisation algorithm. Sim et al. (1997) implemented multiobjective optimisation for a permanent magnet motor design using a modified genetic algorithm. The genetic algorithm used in this case was adjusted to the vector optimisation problem. Multiobjective optimisation of an interior permanent magnet synchronous motor was carried out again by Sim et al. (Cho et al. 1999). In both cases the authors chose weight of the motor and the loss as objective functions. In the present work the MOOP of PM motors is taken a step further. The optimisation of the motor so far laid focus mainly on the magnetic circuit of the motor. In this work a methodology is presented for design of BLDC motor drive. The BLDC motor drive considered here consists of the BLDC motor and voltage source inverter. Here the entire system is considered while designing the motor and hence the system design approach is used to design the BLDC motor.

In this section the PDM is applied for the design of a BLDC motor for a specific application. All the steps of PDM are applied and the motor is designed that optimal with respect to the system in which it has to work. In the next subsection the customer requirements are elicited and validated.

6.1 System requirement analysis

The specified parameters of the motor are:

| | |
|------------------|----------------------|
| Rated speed | 800 rpm (mechanical) |
| Torque at speed | 0.2 Nm |
| Input voltage | 24 V |
| Number of phases | 3 |

The aim of the problem is to design a motor with a cogging torque of less than 20 mNm, maximum efficiency, and minimum mass and trapezoidal back emf.

| | |
|------------------|--|
| Inverter | Full bridge voltage source inverter |
| Motor topology | Inner rotor with surface mount magnets |
| Phase connection | The phases are connected in star |

The additional constraints of the motor are

| | |
|-----------------------|--------|
| Outer stator diameter | 40 mm |
| Max. length | 50 mm |
| Air gap length | 0.2 mm |

6.2 Definition of system boundaries

The BLDC motor to be designed is driven by a voltage source inverter (VSI) and a fixed voltage of 24 V. Hence while designing the motor it is important to include the VSI in the system boundaries. This will ensure that the designed motor will produce the required torque when it is integrated with the VSI. During the design of the motor the parameters of the VSI itself will not be optimised. In the current scenario motor is the primary system under investigation. The model of the system that includes the BLDC motor and the VSI is more complicated but will ensure a well designed motor. Hence the system boundary under consideration in the *synthesis* phase consists of:

- The BLDC motor (primary system),
- three phase VSI.

6.3 Determining of performance criteria

From the requirement analysis the primary objectives that have to be satisfied are:

- Minimum cogging torque,
- maximum efficiency,
- minimum mass,
- sinusoidal shape of back EMF.

In the *synthesis* phase of PDM only simple model of the BLDC drive is developed. However determining parameters like cogging torque and shape of the back emf required detailed analytical models or FEM models. The mass and efficiency of the motor can be calculated with relative ease compared to the cogging torque and back emf shape. Hence in the *synthesis* phase the objectives that will be considered are

- Minimise the mass,
- maximise the efficiency.

A generic topology of BLDC motor with surface mount magnets as shown in Fig. 10 are considered. This topology is optimised for minimum mass and maximum efficiency. In the final design the parameters of this optimised generic topology are fine-tuned to reduce the cogging torque and obtain sinusoidal back emf shape.

6.4 Selection of variables and sensitivity analysis

The following set of *independent design variables* are identified

- Number of poles (N_p),
- number of slots (N_m),
- length of the motor (L_{mot}),
- ratio of inner diameter of motor to outer diameter (α_d),

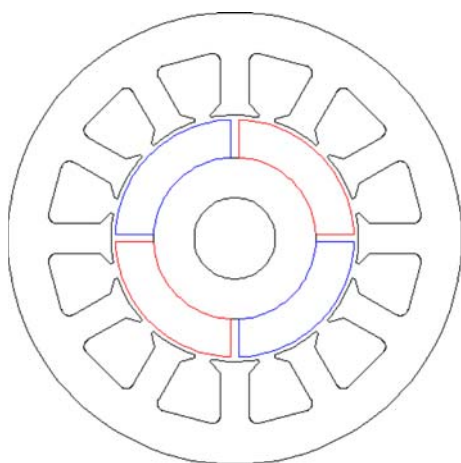


Fig. 10 The generic topology of the BLDC motor with surface mount magnets

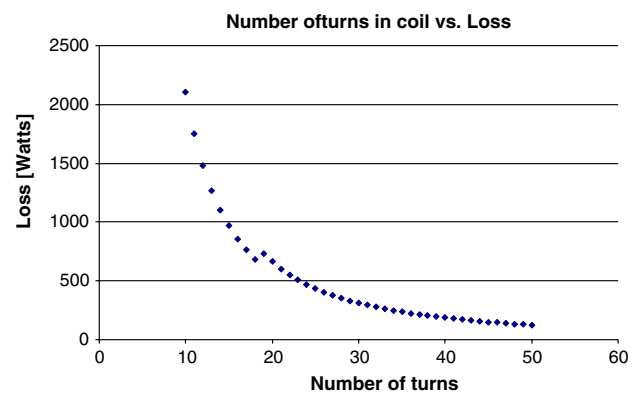


Fig. 11 Sensitivity of loss w.r.t. number of turns the in the coil

- ratio of magnet angle to pole pitch (α_m),
- height of the magnet (h_m),
- remanance field of the permanent magnets (B_r),
- maximum allowable field density in the lamination material for linear operation (B_{fe}),
- number of turns in the coils of the motor (N_{turns}).

Sensitivity analysis is performed to determine influence of the *engineering variables* on the *objectives* viz. mass and efficiency. In Figs. 11 to 24 the sensitivity curves of losses and mass w.r.t. single design variables are shown. From Fig. 11 it can be seen that as the number of turns in the coil increases the losses in the motor decrease. This is due to the fact that with higher number of turns the induced back emf increases as a result of this the difference between the input voltage (24 V in the present case) and the back emf reduces thereby reducing the magnitude of the phase current and hence the Ohmic losses, proportional to square of the current, reduce. The losses of the motor are also sensitive to length of the motor (length of the motor is same as length of the magnet in the present analysis) and reach a minimum value as the length increases Fig. 12. The ratio of inner diameter to outer diameter of the stator has an influence on the losses in the motor, Fig. 13. As the ratio increases the losses reduce because the stator yoke is thicker and as a result of this the field density in the yoke is less resulting in reduction of eddy current and hysteresis losses. As the ratio of magnet angle to pole pitch increases the losses reduces, Fig. 14. A smaller ratio of magnet angle to pole pitch results in smaller magnet and hence less field density in the iron part, thereby reducing the eddy and hysteresis losses in the iron part of the motor. The remanance field of the permanent magnet and maximum allowable field density in iron for linear characteristics have influence on the losses, Figs. 15 and 16, respectively. The height of the magnet also influences the losses in the motor, Fig. 17.

The mass of the motor more or less remaining constant with increase in the number of turns of the coil, Fig. 18.

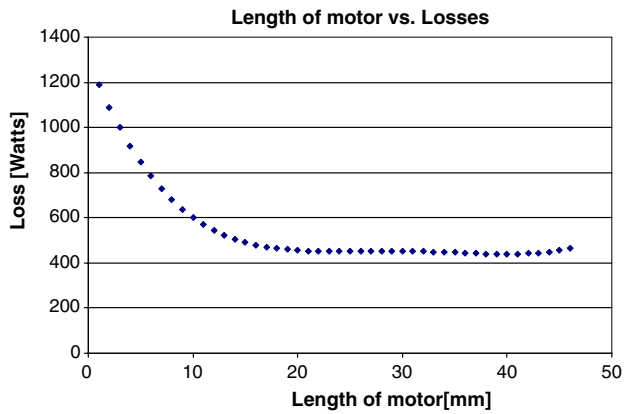


Fig. 12 Sensitivity of loss w.r.t. length of motor

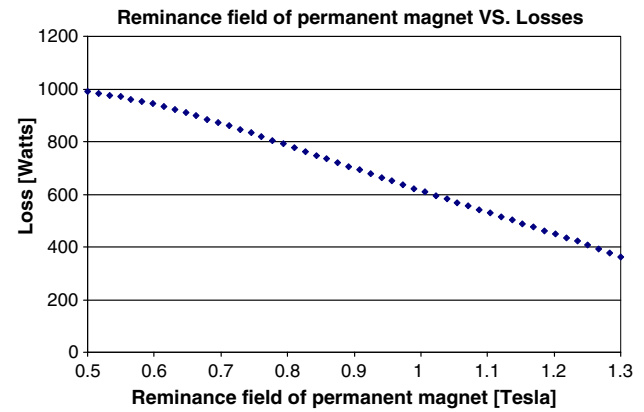


Fig. 15 Sensitivity of loss w.r.t. remanance field allowable Density of permanent magnet characteristics

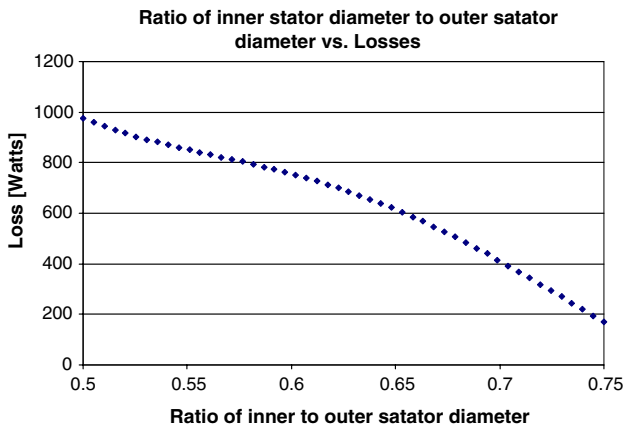


Fig. 13 Sensitivity of loss w.r.t. ratio of inner to outer stator diameters

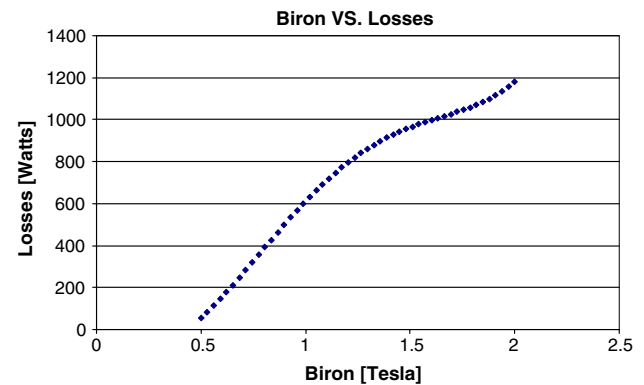


Fig. 16 Sensitivity of loss w.r.t. maximum field density in iron for linear

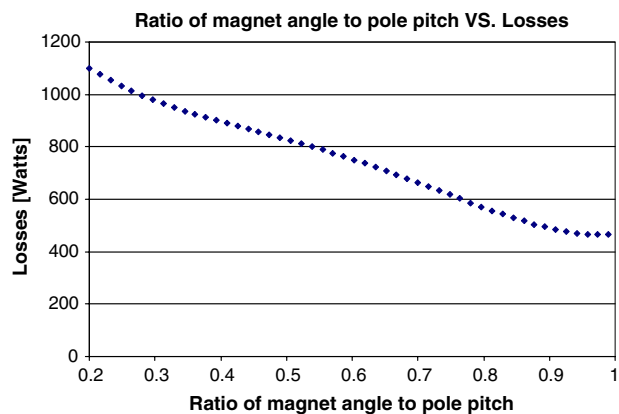


Fig. 14 Sensitivity of loss w.r.t. ratio of magnet angle to pole pitch

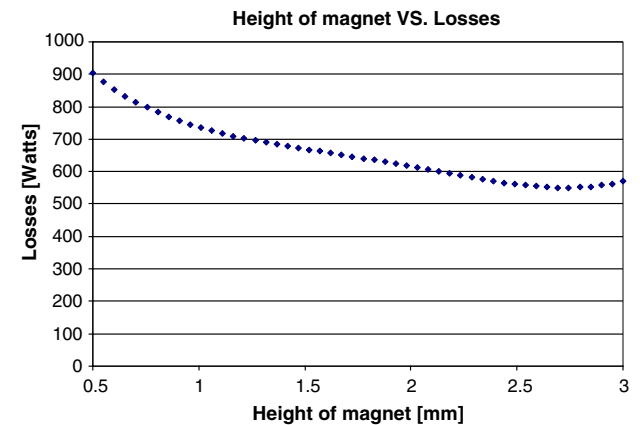


Fig. 17 Sensitivity of loss w.r.t. height of magnet

This is due to the fact that the slot fill ratio is kept constant in the present analysis, hence increase in the number turns does have an influence on the total mass of copper. The mass is directly proportional to motor length, Fig. 19. As the ratio of stator inner and outer diameter increases the

mass of the motor reduces because the amount of iron in the stator reduces, Fig. 20. The mass reach a maximum as the ratio of the magnet angle to the pole pitch increases, Fig. 21. The influence of remanance field density of the permanent magnet on the mass of the motor is shown in

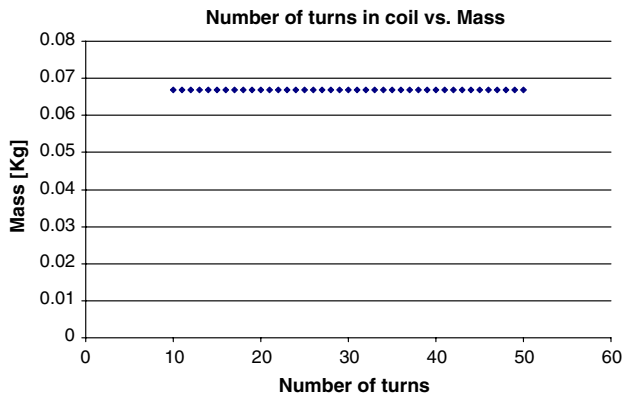


Fig. 18 Sensitivity of mass w.r.t. number of turns

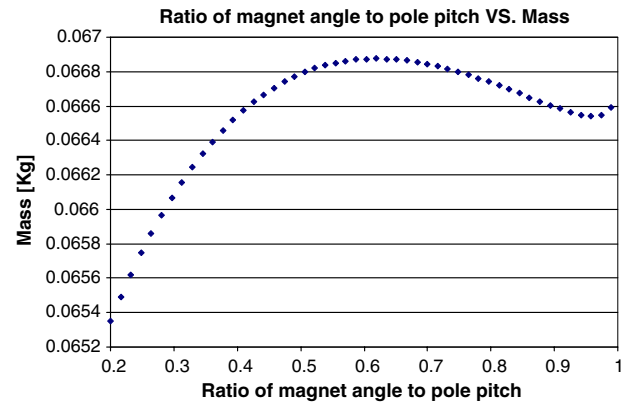


Fig. 21 Sensitivity of mass w.r.t. ratio of magnet to pole pitch

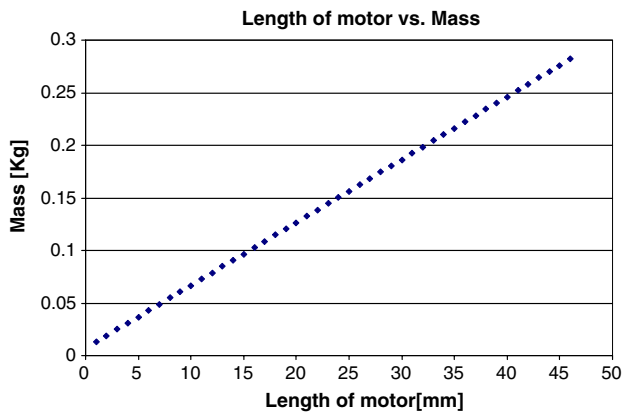


Fig. 19 Sensitivity of mass w.r.t. motor length

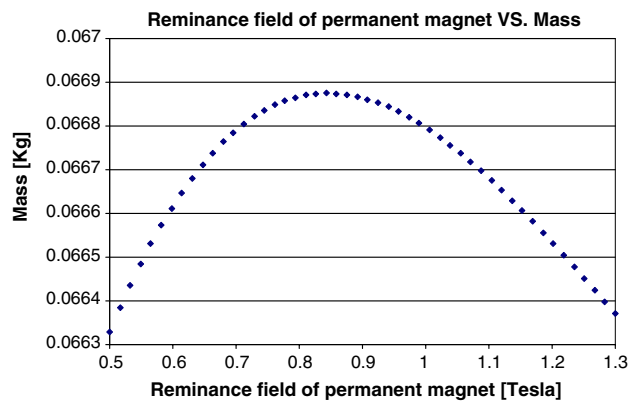


Fig. 22 Sensitivity of mass w.r.t. reminance field density of permanent magnet

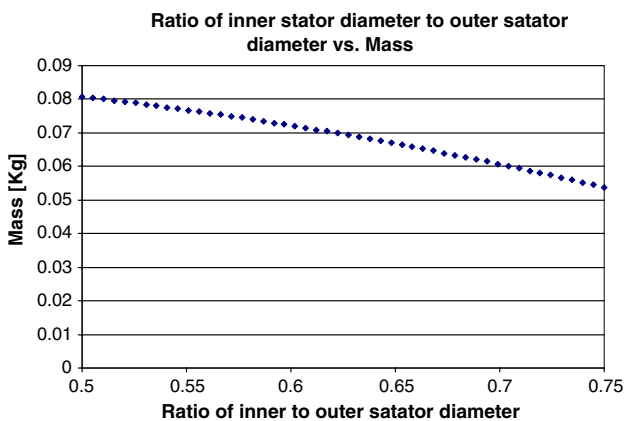


Fig. 20 Sensitivity of mass w.r.t. ratio of stator inner and outer diameter

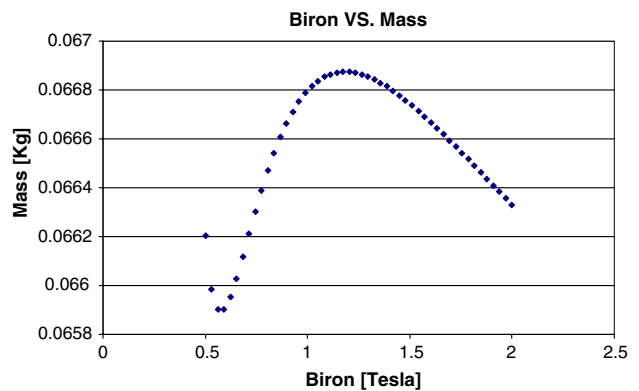


Fig. 23 Sensitivity of mass w.r.t. maximum allowable field density in iron for linear characteristics

Fig. 22. Similarly the influence of maximum allowable field density in iron for linear characteristics on motor mass is shown in Fig. 23. The height of the magnet has influence on the mass as shown in Fig. 24.

The result of the sensitivity analysis show that the selected *engineering* variable have an influence on the

objectives (mass and losses) selected for synthesis analysis. After having performed sensitivity analysis the next step is to determine the optimisation strategy and set up the problem for optimisation. These steps are described in next subsection.

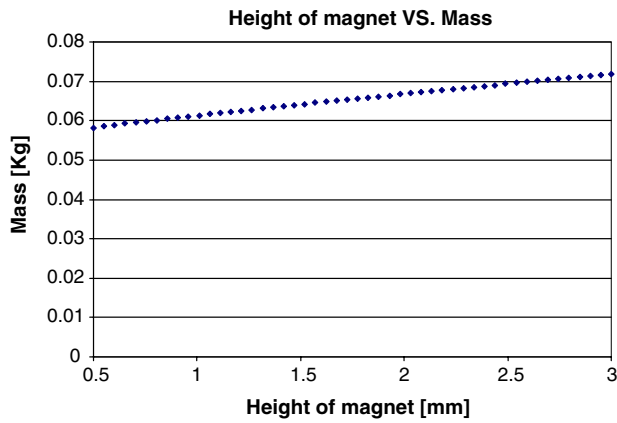


Fig. 24 Sensitivity of mass w.r.t. height of magnet

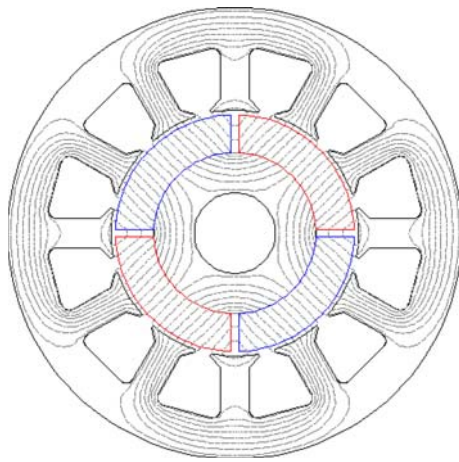


Fig. 25 Flux distribution in a typical BLDC motor

6.5 Development of system model

6.5.1 Motor model

In this section a simple design methodology for the surface mounted BLDC motor is given (Hanselmann 2003). To develop this model certain assumptions have been made. The assumptions made are:

- No saturation in iron parts,
- magnets are symmetrically placed,
- slots are symmetrically placed,
- back emf is trapezoidal in shape,
- motor has balanced windings,
- permeability of iron is infinite.

The general configuration of the motor is shown in Fig. 28. The motor design equations are developed in the following sequence:

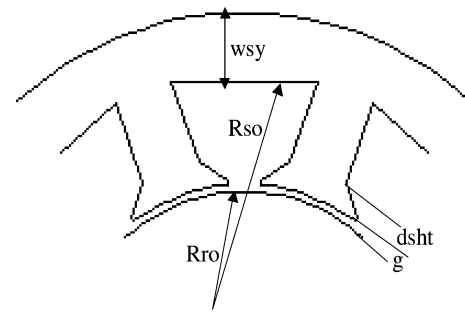


Fig. 26 The general shape of the slots and main dimensions

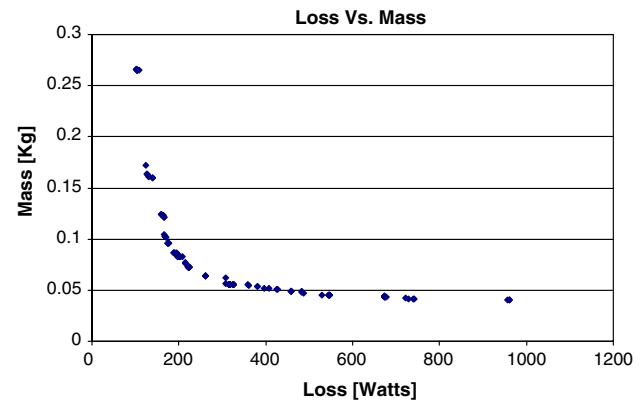


Fig. 27 Pareto optimal solutions for a BLDC motor with $N_s = 6$ and $N_p = 4$

- Electrical design,
- general sizing,
- inductance and resistance calculation.

6.5.1.1 Electrical design (back emf and torque) The back emf voltage induced in a stator coil due to magnet flux crossing the air gap is given by

$$e_{ph} = \frac{d\lambda}{dt} = \frac{d\theta}{dt} \frac{d\lambda}{d\theta} = \omega_m \frac{d\lambda}{d\theta} \quad (1)$$

where ω_m is the mechanical speed of the rotor (radians/s) and λ is the flux linked by the coil.

The magnitude of back emf is given by

$$|e_{ph}| = 2B_g N_{turns} L_{stack} R_{ro} \omega_m \quad (2)$$

where N_{turns} is the number of turns in a coil, L_{stack} is the length rotor of the stack, R_{ro} is the outer radius of the B_g is the air gap field density given by

$$B_g = B_r \frac{h_m}{h_m + g} \quad (3)$$

where h_m is the height of the magnet, g is the airgap and the outer radius of the rotor R_{ro} is given by

$$R_{ro} = \alpha_d R_o \quad (4)$$

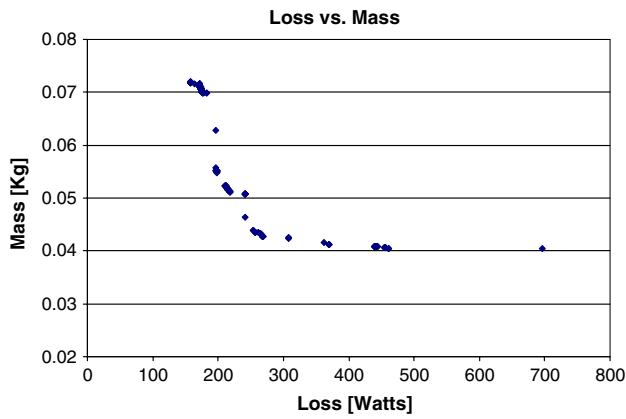


Fig. 28 Pareto optimal solutions for a BLDC motor with $N_s = 9$ and $N_p = 6$

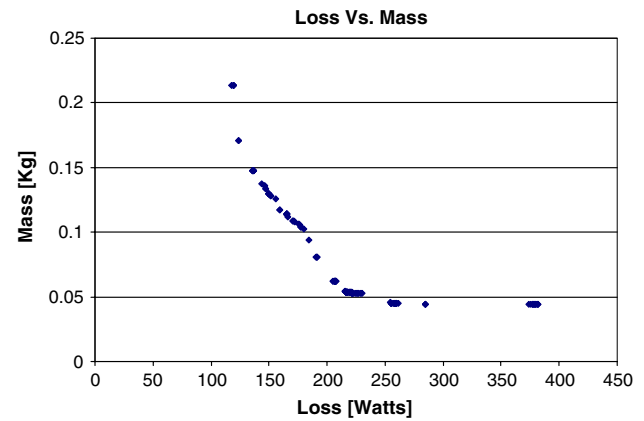


Fig. 30 Pareto optimal solutions for a BLDC motor with $N_s = 9$ and $N_p = 10$

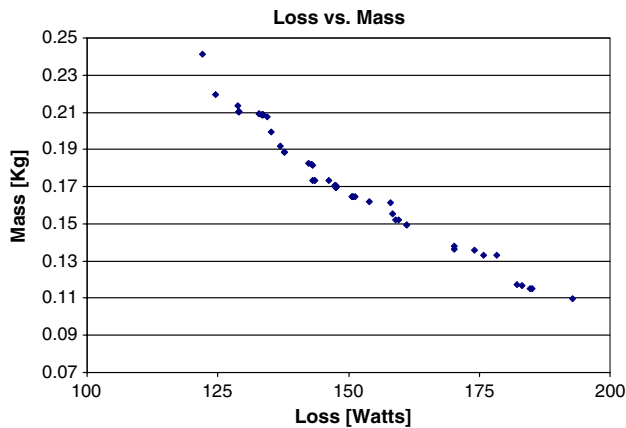


Fig. 29 Pareto optimal solutions for a BLDC motor with $N_s = 9$ and $N_p = 8$

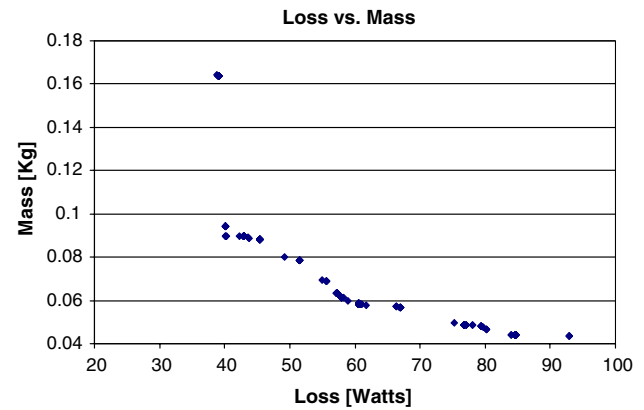


Fig. 31 Pareto optimal solutions for a BLDC motor with $N_s = 12$ and $N_p = 8$

where the R_o is outer radius of the stator and α_d is the ratio of outer diameter of the rotor to the outer diameter of the stator.

For fractional pitched magnets the coil back emf is given by

$$e_{ph} = 2\alpha_m B_g N_{turns} L_{stack} R_{ro} \omega_m \quad (5)$$

6.5.1.2 General sizing If the alternating direction of flux flow over alternating magnet faces is ignored, the total flux crossing the air gap is given by

$$\phi_{total} = B_g A_g = 2\pi B_g R_{ro} L_{stack} \quad (6)$$

where B_g is the amplitude of the air gap flux density and is given by (9) and A_g is area of the airgap. This flux is divided among the teeth on the stator and the direction of the flux depends on the polarity of the magnet under each tooth. As a result, the magnitude of the flux flowing in each tooth is given by

$$\phi_t = \frac{\phi_{total}}{N_s} = \frac{2\pi B_g R_{ro} L_{stack}}{N_s} \quad (7)$$

where N_s is the number of slots on the stator.

This flux travels through the body of the tooth resulting in a flux density B_t whose magnitude is given by

$$B_t = \frac{\phi_t}{w_{tb} L_{stack}} \quad (8)$$

where w_{tb} is the width of the tooth.

The value of B_t is generally known, as it is the maximum allowable flux density in iron. Hence once the value of B_t is determined the width of the tooth is given from Eqs. 7 and 8

$$w_{tb} = \frac{2\pi R_{ro} B_g}{N_s B_t} \quad (9)$$

From the above expression it can be seen that the tooth width is directly dependent on the rotor outer radius and inversely proportional to number of slots. As the number of

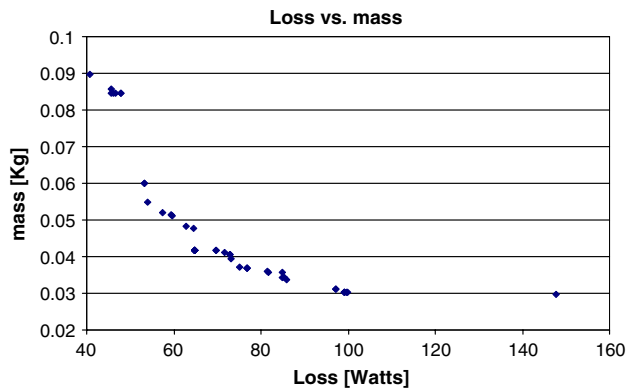


Fig. 32 Pareto optimal solutions for a BLDC motor with $N_s = 15$ and $N_p = 10$

slot increase, the width of the tooth decreases. The width of the tooth is independent of the number of poles (magnets) because the total flux crossing the air gap is not a function of the number of poles. Hence w_{tb} does not vary with the change in the number of poles.

The flux from each magnet splits into two halves, with each half forming a flux loop, Fig. 14. The stator flux density is B_{sy} given by

$$B_{sy} = \frac{\phi_{total}/2}{w_{sy}L_{stack}} \quad (10)$$

where w_{sy} is the width of the yoke and is given by

$$w_{sy} = \frac{\pi R_{ro} B_g}{N_m B_{sy}} \quad (11)$$

B_{sy} is again the maximum permissible flux density in iron. The above expression shows that width of the yoke is directly proportional to the outer radius of the rotor R_{ro} and inversely proportional to the number of magnets. The stator yoke width is independent of number of slots.

The general shape of the slot considered here is shown in Figs. 25, 26.

The area of the above slot is given by

$$A_s = \frac{\pi}{N_s} \left[(R_{so} - w_{sy})^2 - (R_{ro} + g + d_{sht})^2 \right] - w_{tb} (R_{so} - w_{sy} - R_{ro} - g - d_{sht}) \quad (12)$$

The area of the copper in the slot is given by

$$A_{cu} = K_{cu} A_s \quad (13)$$

where k_{cu} is the copper fill factor.

With this the calculation of the main dimensions of the motor is done.

6.5.1.3 Inductance and resistance calculation The total phase inductance L_{ph} composed of air gap inductance L_g ,

Table 4 Importance of different parameters used in screening process

| Parameter | Importance | Direction |
|--|------------|-----------|
| Length of the stack | M | L |
| Losses | H | L |
| Mass | VH | L |
| Electrical time constant | H | L |
| Inertia of the rotor | L | L |
| Ratio of inner stator to outer stator diameters | H | H |
| Number of turns | M | L |
| Reminance field of permanent magnet | N | L |
| Max. field density in stator lamination material | N | L |
| Width of the tooth | VL | L |
| Width of the yoke | L | L |
| Area of slots | H | H |

slot leakage inductance L_s , and end turn inductance L_e is given (Hanselmann 2003)

$$L_{ph} = N_s/3(L_s + L_e + L_g) \quad (14)$$

where

$$L_g = \frac{N_{turns}^2 \mu_r \mu_o L_{stack} \tau_c k_d}{4(L_{stack} + \mu_r k_c g)} \quad (15)$$

$$L_s = N_{turns}^2 \left[\frac{\mu_o d_3^2 L_{stack}}{3A_s} + \frac{\mu_o d_2 L_{stack}}{(w_s + w_{si})/2} + \frac{\mu_o d_1 L_{stack}}{w_s} \right] \quad (16)$$

$$L_e = \frac{\mu_o N_{turn}^2 \tau_c}{8} \ln \left[\frac{\tau_c^2 \pi}{4A_s} \right] \quad (17)$$

where μ_o is the permeability of free space.

The parameters of the motor are shown in Fig. 26. Since each slot has two coil sides and each coil has N_{turns} turns, the resistance R_{slot} per slot is given by

$$R_{slot} = \frac{\rho L_{stack} 4N_{turns}^2}{K_{cu} A_{slot}} \quad (18)$$

A three phase star connected motor is considered. Hence the phase resistance of the motor is given by

$$R_{ph} = \frac{N_s}{3} R_{slot} \quad (19)$$

6.5.1.4 Loss calculation The Ohmic P_r and the core loss P_{Fe} can be determined from the following relation:

$$P_r = 3I_{ph}^2 R_{ph} \quad (20)$$

$$P_{Fe} = \rho_{bi} V_{st} \Gamma(f_c, B_{Fe}) \quad (21)$$

where I_{ph} is the RMS value of the driving current for each phase, R_{ph} is the resistance value for each phase, ρ_{bi} is the mass density of the back iron, V_{st} is the stator volume and

Table 5 The results of screening for the motor with $N_s = 6$ and $N_p = 4$

| Direction Importance Alternatives | L M L_{stack} | L H Losses | L VH Mass | L H Time constant | L L Inertia | H H DiDo | L M N_{turns} | L VL wt | L L wy | H H Aslots | Final value |
|---|-----------------------|------------------|-----------------|-------------------------|-------------------|----------------|-----------------------|---------------|--------------|------------------|-------------|
| 1 | 39.080 | 102.921 | 0.266 | 0.0002397 | 5.058 | 0.600 | 59 | 7.083 | 5.312 | 3.111 | VL |
| 2 | 39.075 | 103.171 | 0.266 | 0.0002412 | 4.993 | 0.600 | 59 | 7.071 | 5.303 | 3.111 | VL |
| 3 | 39.076 | 109.431 | 0.265 | 0.0002846 | 4.860 | 0.600 | 59 | 6.924 | 5.193 | 3.164 | VL |
| 4 | 23.731 | 124.825 | 0.172 | 0.0002911 | 3.710 | 0.581 | 59 | 7.093 | 5.320 | 3.203 | L |
| 5 | 23.778 | 125.444 | 0.164 | 0.0002917 | 3.035 | 0.595 | 60 | 6.805 | 5.104 | 3.299 | L |
| 6 | 23.789 | 127.760 | 0.163 | 0.0002988 | 3.087 | 0.600 | 59 | 6.688 | 5.016 | 3.356 | L |
| 7 | 23.789 | 128.071 | 0.163 | 0.0003007 | 3.082 | 0.600 | 59 | 6.681 | 5.011 | 3.363 | L |
| 8 | 22.637 | 129.909 | 0.161 | 0.0003286 | 2.828 | 0.580 | 60 | 6.929 | 5.197 | 3.317 | L |
| 9 | 22.664 | 139.730 | 0.160 | 0.0003093 | 2.673 | 0.581 | 53 | 6.993 | 5.245 | 3.263 | L |
| 10 | 17.241 | 159.536 | 0.124 | 0.0003367 | 2.305 | 0.581 | 59 | 6.714 | 5.036 | 3.554 | L |
| 11 | 17.174 | 162.847 | 0.123 | 0.0003636 | 2.101 | 0.581 | 60 | 6.602 | 4.952 | 3.723 | L |
| 12 | 17.174 | 166.638 | 0.121 | 0.0003473 | 2.577 | 0.599 | 60 | 6.339 | 4.755 | 3.901 | L |
| 13 | 14.710 | 167.018 | 0.104 | 0.0003287 | 2.110 | 0.599 | 59 | 6.304 | 4.728 | 3.972 | L |
| 14 | 14.439 | 170.149 | 0.102 | 0.0003425 | 1.931 | 0.599 | 59 | 6.232 | 4.674 | 4.122 | L |
| 15 | 13.216 | 174.275 | 0.096 | 0.0003213 | 2.216 | 0.600 | 59 | 6.231 | 4.673 | 4.098 | L |
| 16 | 13.216 | 176.200 | 0.095 | 0.0003275 | 2.162 | 0.600 | 59 | 6.204 | 4.653 | 4.156 | L |
| 17 | 11.973 | 188.203 | 0.086 | 0.0003321 | 1.805 | 0.600 | 59 | 6.101 | 4.576 | 4.399 | L |
| 18 | 11.973 | 189.799 | 0.086 | 0.000332 | 1.805 | 0.600 | 59 | 6.101 | 4.575 | 4.399 | L |
| 19 | 11.991 | 194.382 | 0.086 | 0.0003491 | 1.678 | 0.600 | 59 | 6.027 | 4.520 | 4.587 | L |
| 20 | 11.451 | 196.572 | 0.083 | 0.0003423 | 1.728 | 0.600 | 59 | 6.014 | 4.511 | 4.620 | L |
| 21 | 11.433 | 200.977 | 0.083 | 0.0003562 | 1.682 | 0.599 | 59 | 5.971 | 4.478 | 4.769 | L |
| 22 | 11.447 | 201.833 | 0.083 | 0.0003556 | 1.676 | 0.600 | 59 | 5.954 | 4.466 | 4.784 | L |
| 23 | 11.447 | 203.203 | 0.083 | 0.0003611 | 1.611 | 0.600 | 59 | 5.930 | 4.448 | 4.852 | L |
| 24 | 11.429 | 207.990 | 0.082 | 0.0003738 | 1.555 | 0.600 | 59 | 5.873 | 4.404 | 5.020 | L |
| 25 | 10.437 | 214.082 | 0.077 | 0.0003576 | 1.544 | 0.599 | 59 | 5.878 | 4.408 | 5.044 | L |
| 26 | 10.450 | 215.020 | 0.076 | 0.0003645 | 1.396 | 0.599 | 59 | 5.847 | 4.385 | 5.137 | L |
| 27 | 9.858 | 221.809 | 0.073 | 0.0003605 | 1.355 | 0.595 | 59 | 5.861 | 4.396 | 5.188 | L |
| 28 | 9.817 | 222.447 | 0.073 | 0.000363 | 1.289 | 0.595 | 59 | 5.847 | 4.385 | 5.236 | L |
| 29 | 9.785 | 223.964 | 0.072 | 0.0003632 | 1.375 | 0.600 | 59 | 5.760 | 4.320 | 5.370 | L |
| 30 | 8.172 | 261.767 | 0.064 | 0.0003974 | 0.998 | 0.581 | 59 | 5.701 | 4.276 | 6.147 | L |
| 31 | 8.148 | 306.782 | 0.062 | 0.000323 | 1.262 | 0.600 | 49 | 5.760 | 4.320 | 5.381 | L |
| 32 | 6.516 | 307.858 | 0.056 | 0.0003963 | 0.956 | 0.570 | 59 | 5.544 | 4.158 | 7.069 | L |
| 33 | 6.510 | 313.504 | 0.056 | 0.0004111 | 0.877 | 0.570 | 59 | 5.461 | 4.096 | 7.430 | L |
| 34 | 6.517 | 316.436 | 0.056 | 0.0004174 | 0.838 | 0.570 | 59 | 5.428 | 4.071 | 7.579 | L |
| 35 | 6.507 | 318.500 | 0.056 | 0.0004168 | 0.836 | 0.570 | 59 | 5.429 | 4.072 | 7.574 | L |
| 36 | 6.523 | 325.143 | 0.056 | 0.0004387 | 0.755 | 0.570 | 59 | 5.316 | 3.987 | 8.103 | L |
| 37 | 6.573 | 358.582 | 0.055 | 0.0003779 | 0.829 | 0.570 | 53 | 5.660 | 4.245 | 6.587 | L |
| 38 | 6.507 | 361.755 | 0.055 | 0.0003759 | 0.807 | 0.570 | 53 | 5.656 | 4.242 | 6.605 | L |
| 39 | 6.583 | 381.327 | 0.054 | 0.0003302 | 0.879 | 0.581 | 47 | 5.765 | 4.324 | 5.908 | M |
| 40 | 6.551 | 396.263 | 0.052 | 0.0003356 | 0.875 | 0.600 | 47 | 5.414 | 4.060 | 6.642 | M |
| 41 | 6.510 | 408.011 | 0.051 | 0.0002838 | 0.956 | 0.600 | 44 | 5.709 | 4.282 | 5.545 | L |
| 42 | 5.069 | 424.016 | 0.051 | 0.0004663 | 0.594 | 0.526 | 59 | 5.355 | 4.016 | 9.121 | L |
| 43 | 5.012 | 428.063 | 0.051 | 0.0004665 | 0.572 | 0.526 | 59 | 5.329 | 3.997 | 9.264 | L |
| 44 | 5.863 | 457.472 | 0.049 | 0.0003225 | 0.708 | 0.581 | 44 | 5.645 | 4.234 | 6.355 | M |

Table 6 The results of screening for the motor with $N_s = 9$ $N_p = 10$

| Direction Importance Alternative | L M L_{stack} | L H Losses | L VH Mass | L H Time constant | L L J_{mot} | H H DiDo | L M N_{turns} | L VL wt | L L wy | H H Aslots | Final value |
|--|------------------------------|------------------|-----------------|-------------------------|----------------------------|----------------|------------------------------|---------------|--------------|------------------|-------------|
| 1 | 31.119 | 117.451 | 0.214 | 0.0026401 | 2.380 | 0.508 | 60 | 3.288 | 1.479 | 32.508 | VL |
| 2 | 24.722 | 124.145 | 0.171 | 0.0027403 | 1.829 | 0.508 | 60 | 2.685 | 1.208 | 38.803 | VL |
| 3 | 20.866 | 136.246 | 0.147 | 0.0025642 | 1.811 | 0.516 | 60 | 2.642 | 1.189 | 38.770 | L |
| 4 | 18.946 | 143.236 | 0.138 | 0.0024483 | 1.492 | 0.508 | 60 | 2.841 | 1.279 | 37.140 | L |
| 5 | 17.614 | 149.605 | 0.130 | 0.002309 | 1.378 | 0.508 | 60 | 3.018 | 1.358 | 35.261 | L |
| 6 | 15.732 | 159.273 | 0.118 | 0.0023229 | 1.240 | 0.508 | 60 | 2.738 | 1.232 | 38.254 | L |
| 7 | 14.713 | 166.044 | 0.111 | 0.0022571 | 1.162 | 0.508 | 60 | 2.740 | 1.233 | 38.202 | L |
| 8 | 13.751 | 175.540 | 0.106 | 0.0021383 | 1.072 | 0.508 | 60 | 2.882 | 1.297 | 36.682 | L |
| 9 | 13.788 | 180.009 | 0.103 | 0.0020036 | 1.270 | 0.537 | 60 | 2.788 | 1.254 | 35.755 | L |
| 10 | 8.046 | 205.460 | 0.062 | 0.0008258 | 1.014 | 0.600 | 60 | 4.478 | 2.015 | 17.609 | L |
| 11 | 8.009 | 207.324 | 0.062 | 0.0008158 | 1.010 | 0.600 | 60 | 4.514 | 2.031 | 17.371 | L |
| 12 | 5.077 | 254.040 | 0.046 | 0.0006926 | 0.651 | 0.599 | 60 | 3.993 | 1.797 | 21.016 | L |

Γ is the core loss density of the stator material at the flux density B_{Fe} and frequency f_c .

6.5.2 Dynamic performance of BLDC motor

The derivation of this model is based on the assumptions that the induced currents in the rotor due to the stator harmonic fields are neglected.

The coupled circuit equations of the stator windings in terms of the motor electrical constants are

$$[V] = [R][i] + [L] \frac{d[i]}{dt} + [e] \quad (22a)$$

$$\text{where } [V] = [V_a, V_b, V_c]' \quad (22b)$$

$$[R] = \begin{bmatrix} R_{\text{ph}} & 0 & 0 \\ 0 & R_{\text{ph}} & 0 \\ 0 & 0 & R_{\text{ph}} \end{bmatrix} \quad (22c)$$

$$[i] = [i_a, i_b, i_c]' \quad (22d)$$

$$[L] = \begin{bmatrix} L_{\text{ph}} & 0 & 0 \\ 0 & L_{\text{ph}} & 0 \\ 0 & 0 & L_{\text{ph}} \end{bmatrix} \quad (22e)$$

$$[e] = [e_a, e_b, e_c]' \quad (22f)$$

where R_{ph} and L_{ph} are the phase resistance and phase inductance values, respectively defined earlier and V_a , V_b , and V_c are the input voltages to each phase a , b and c , respectively. The induced emf e_a , e_b , e_c are sinusoidal in shape and their peak values are given by Eq. 2. The electromagnetic torque is given by

$$T_e = [e_a i_a + e_b i_b + e_c i_c] \frac{1}{\omega_m} \quad (23)$$

where ω_m is the mechanical speed of the motor.

The analytical solution of the Eq. 27a is given by Nucera et al. (Nucera et al. 1989). In this work the model developed by Nucera et al. has been used.

6.6 Optimisation strategy

In the present case study optimisation strategy based on *Posteriori articulation of preference information* is used. To achieve the multiobjective optimisation the Nondominated sorting Biologically Motivated Genetic Algorithm (NBGA) (Kumar et al. 2006) is used. The parameters of NBGA are as follows

Number of generations = 50

Number of individuals = 100

Crossover probability = 80%

Single point crossover was used.

The mutation rate was fixed between 0 and 10%

Hence the multiobjective optimisation problem to be solved is expressed mathematically as

$$\text{minimise} \begin{cases} f_1(\vec{x}) = P_{\text{cu}} + P_{\text{hys}} + P_{\text{eddy}} \\ f_2(\vec{x}) = M_{\text{iron}} + M_{\text{magnet}} \end{cases}$$

where P_{cu} , P_{hys} and P_{eddy} are the copper loss, hysteresis loss in the stator yoke and the eddy current loss in the stator yoke, respectively and M_{iron} and M_{magnet} are the mass of yoke (stator and rotor) and mass of permanent magnets, respectively.

subject to $h(\vec{x}) = T_{\text{motor}} \geq 0.2 \text{ Nm}$

where $\vec{x} = (B_r, B_{\text{Fe}}, h_m, L_{\text{motor}}, \alpha_m, \alpha_d, N_m, N_s, N_{\text{turns}})$ are the independent design variables

Table 7 The results of screening for the motor with $N_s = 9$ $N_p = 6$

| Direction Importance Alternative | L M L_{stack} | L H Losses | L VH Mass | L H Time constant | L L J_{mot} | H H DiDo | L M N_{turns} | L VL wt | L L wy | H H Aslots | Final value |
|--|-----------------------|------------------|-----------------|----------------------------|---------------------|----------------|-----------------------|---------------|--------------|------------------|----------------|
| 1 | 9.786 | 157.634 | 0.072 | 0.0003752 | 1.688 | 0.600 | 60 | 5.054 | 3.790 | 8.263 | VL |
| 2 | 9.786 | 164.114 | 0.072 | 0.000383 | 1.613 | 0.600 | 59 | 5.012 | 3.759 | 8.470 | VL |
| 3 | 9.786 | 171.410 | 0.072 | 0.0003836 | 1.602 | 0.600 | 60 | 5.009 | 3.757 | 8.486 | VL |
| 4 | 9.794 | 172.560 | 0.071 | 0.000397 | 1.486 | 0.600 | 60 | 4.940 | 3.705 | 8.838 | VL |
| 5 | 9.792 | 173.388 | 0.071 | 0.0004052 | 1.421 | 0.600 | 60 | 4.897 | 3.673 | 9.061 | VL |
| 6 | 9.799 | 174.509 | 0.070 | 0.0004174 | 1.334 | 0.600 | 60 | 4.836 | 3.627 | 9.386 | VL |
| 7 | 9.804 | 175.171 | 0.070 | 0.0004244 | 1.288 | 0.600 | 60 | 4.801 | 3.601 | 9.576 | VL |
| 8 | 9.786 | 177.536 | 0.070 | 0.0004294 | 1.292 | 0.600 | 60 | 4.773 | 3.580 | 9.727 | VL |
| 9 | 9.786 | 182.972 | 0.070 | 0.0004283 | 1.291 | 0.600 | 59 | 4.779 | 3.584 | 9.695 | VL |
| 10 | 9.786 | 183.042 | 0.070 | 0.0004283 | 1.291 | 0.600 | 59 | 4.779 | 3.584 | 9.695 | VL |
| 11 | 8.427 | 196.408 | 0.063 | 0.0004116 | 1.381 | 0.600 | 60 | 4.680 | 3.510 | 10.254 | L |
| 12 | 7.344 | 197.133 | 0.055 | 0.0004021 | 1.033 | 0.600 | 60 | 4.548 | 3.411 | 11.035 | L |
| 13 | 7.331 | 198.013 | 0.055 | 0.0004066 | 0.995 | 0.600 | 60 | 4.520 | 3.390 | 11.204 | L |
| 14 | 6.667 | 211.152 | 0.052 | 0.0003902 | 1.150 | 0.600 | 60 | 4.478 | 3.358 | 11.467 | L |
| 15 | 6.641 | 213.407 | 0.052 | 0.0004011 | 1.052 | 0.600 | 60 | 4.409 | 3.307 | 11.897 | L |
| 16 | 6.650 | 215.335 | 0.052 | 0.0004138 | 0.965 | 0.600 | 60 | 4.340 | 3.255 | 12.347 | L |
| 17 | 6.642 | 218.685 | 0.051 | 0.0004337 | 0.849 | 0.600 | 60 | 4.228 | 3.171 | 13.092 | L |
| 18 | 6.641 | 218.706 | 0.051 | 0.0004337 | 0.848 | 0.600 | 60 | 4.228 | 3.171 | 13.095 | L |
| 19 | 6.659 | 241.040 | 0.051 | 0.0003775 | 0.936 | 0.600 | 52 | 4.550 | 3.413 | 11.021 | L |
| 20 | 5.231 | 253.763 | 0.044 | 0.0003926 | 0.933 | 0.600 | 58 | 4.085 | 3.063 | 14.091 | L |
| 21 | 5.231 | 254.492 | 0.044 | 0.0003927 | 0.932 | 0.600 | 58 | 4.084 | 3.063 | 14.095 | L |
| 22 | 5.146 | 257.163 | 0.043 | 0.0003925 | 0.917 | 0.600 | 58 | 4.058 | 3.043 | 14.282 | L |
| 23 | 5.176 | 261.857 | 0.043 | 0.0003957 | 0.867 | 0.600 | 60 | 4.048 | 3.036 | 14.353 | L |
| 24 | 5.191 | 264.120 | 0.043 | 0.0004153 | 0.753 | 0.600 | 60 | 3.934 | 2.950 | 15.191 | L |
| 25 | 5.191 | 266.084 | 0.043 | 0.0004225 | 0.705 | 0.600 | 60 | 3.891 | 2.918 | 15.513 | L |
| 26 | 5.155 | 267.243 | 0.043 | 0.0004205 | 0.696 | 0.600 | 60 | 3.891 | 2.918 | 15.513 | L |
| 27 | 5.137 | 269.628 | 0.043 | 0.0004277 | 0.678 | 0.600 | 60 | 3.842 | 2.882 | 15.885 | L |
| 28 | 5.146 | 307.658 | 0.042 | 0.0003999 | 0.699 | 0.600 | 52 | 4.013 | 3.010 | 14.609 | L |
| 29 | 5.138 | 362.249 | 0.042 | 0.0003071 | 0.806 | 0.600 | 42 | 4.619 | 3.464 | 10.609 | L |
| 30 | 5.150 | 370.205 | 0.041 | 0.000323 | 0.699 | 0.600 | 42 | 4.512 | 3.384 | 11.253 | L |
| 31 | 5.150 | 370.205 | 0.041 | 0.000323 | 0.699 | 0.600 | 42 | 4.512 | 3.384 | 11.253 | L |
| 32 | 5.144 | 439.331 | 0.041 | 0.0002639 | 0.780 | 0.600 | 34 | 4.939 | 3.704 | 8.840 | L |

and $0.5 \leq B_r \leq 1.2$, $0.5 \leq B_{Fe} \leq 2$

$$1 \leq h_m \leq 3, 1 \leq L_{motor} \leq 100$$

$$0.1 \leq \alpha_m \leq 1, 0.1 \leq \alpha_d \leq 0.7$$

$$2 \leq N_m \leq 10, 3 \leq N_s \leq 15$$

$$1 \leq N_{turns} \leq 100$$

The results of the optimisation are given the next subsection.

6.7 Results of multiobjective optimisation

The results of optimisation are given in Figs. 27, 28, 29, 30, 31, 32. From the results it can be seen that for each pole

slot combination a number of Pareto optimal solutions are present and as the mass of the motor increases the losses decreases. Since the number of feasible solutions is large the results have to be screened so that a reduced set is obtained. Detailed analysis can be then performed on the reduced set. In the next section the screening process is performed.

7 Intermediate analysis phase of progressive design methodology for design of a BLDC motor drive

In this section the results obtained from the multi-criteria multiobjective optimisation obtained in the previous

Table 8 The results of screening for the motor with $N_s = 9$ $N_r = 8$

| Direction Importance Alternative | L M L _{stack} | L H Losses | L VH Mass | L H Time constant | L L J _{mot} | H H DiDo | L M N _{turns} | L VL wt | L L wy | H H Aslots | Final value |
|--|------------------------------|------------------|-----------------|-------------------------|----------------------------|----------------|------------------------------|---------------|--------------|------------------|-------------|
| 1 | 36.776 | 122.016 | 0.241 | 0.0029401 | 2.736 | 0.510 | 60 | 2.519 | 1.417 | 38.424 | VL |
| 2 | 34.264 | 124.545 | 0.219 | 0.0031552 | 2.813 | 0.518 | 60 | 1.929 | 1.085 | 44.894 | VL |
| 3 | 32.803 | 128.750 | 0.214 | 0.0029982 | 2.816 | 0.521 | 60 | 2.116 | 1.190 | 42.405 | VL |
| 4 | 32.876 | 132.891 | 0.209 | 0.0030011 | 2.650 | 0.528 | 59 | 2.008 | 1.129 | 43.131 | VL |
| 5 | 32.876 | 133.476 | 0.209 | 0.0029885 | 2.668 | 0.530 | 59 | 2.012 | 1.132 | 42.975 | VL |
| 6 | 32.876 | 134.417 | 0.208 | 0.0028896 | 2.857 | 0.542 | 59 | 2.021 | 1.137 | 41.886 | VL |
| 7 | 31.646 | 135.148 | 0.199 | 0.0029401 | 3.238 | 0.542 | 59 | 1.878 | 1.056 | 43.527 | VL |
| 8 | 29.731 | 137.036 | 0.192 | 0.0026618 | 2.891 | 0.542 | 60 | 2.321 | 1.305 | 38.398 | VL |
| 9 | 26.491 | 142.312 | 0.182 | 0.0028708 | 1.840 | 0.501 | 60 | 2.347 | 1.320 | 41.122 | L |
| 10 | 26.574 | 143.091 | 0.182 | 0.0028883 | 1.740 | 0.501 | 59 | 2.318 | 1.304 | 41.472 | L |
| 11 | 25.288 | 146.151 | 0.173 | 0.0029022 | 1.658 | 0.501 | 59 | 2.212 | 1.244 | 42.744 | L |
| 12 | 24.557 | 150.596 | 0.165 | 0.0028077 | 2.036 | 0.521 | 59 | 2.062 | 1.160 | 43.054 | L |
| 13 | 24.673 | 151.189 | 0.164 | 0.0028218 | 2.134 | 0.521 | 60 | 2.039 | 1.147 | 43.321 | L |
| 14 | 24.635 | 153.936 | 0.162 | 0.0026879 | 2.286 | 0.541 | 60 | 2.006 | 1.129 | 42.070 | L |
| 15 | 26.522 | 158.045 | 0.162 | 0.0023082 | 2.768 | 0.595 | 60 | 2.061 | 1.159 | 36.646 | L |
| 16 | 22.448 | 159.025 | 0.152 | 0.0027394 | 1.963 | 0.525 | 59 | 1.997 | 1.123 | 43.552 | L |
| 17 | 22.440 | 161.063 | 0.150 | 0.0026146 | 2.082 | 0.541 | 60 | 2.005 | 1.128 | 42.091 | L |
| 18 | 19.888 | 170.183 | 0.138 | 0.0026272 | 1.821 | 0.521 | 60 | 2.063 | 1.160 | 43.030 | L |
| 19 | 19.772 | 174.089 | 0.136 | 0.0024361 | 1.914 | 0.542 | 60 | 2.154 | 1.212 | 40.312 | L |
| 20 | 19.874 | 175.847 | 0.133 | 0.0023942 | 1.837 | 0.553 | 60 | 2.090 | 1.176 | 40.139 | L |
| 21 | 19.874 | 178.268 | 0.133 | 0.0024049 | 1.836 | 0.553 | 60 | 2.068 | 1.163 | 40.394 | L |
| 22 | 17.181 | 182.271 | 0.117 | 0.0015246 | 1.579 | 0.595 | 60 | 3.204 | 1.802 | 25.286 | L |
| 23 | 17.283 | 183.240 | 0.117 | 0.0015224 | 1.558 | 0.595 | 60 | 3.219 | 1.811 | 25.153 | L |
| 24 | 17.252 | 184.765 | 0.115 | 0.0015063 | 1.450 | 0.600 | 60 | 3.191 | 1.795 | 25.085 | L |
| 25 | 17.216 | 185.155 | 0.115 | 0.0015052 | 1.448 | 0.600 | 60 | 3.190 | 1.794 | 25.089 | L |
| 26 | 16.087 | 192.939 | 0.110 | 0.0015347 | 1.391 | 0.597 | 59 | 3.048 | 1.715 | 26.571 | L |
| 27 | 8.241 | 198.313 | 0.063 | 0.000679 | 1.129 | 0.599 | 60 | 4.679 | 2.632 | 13.932 | L |
| 28 | 8.213 | 199.209 | 0.062 | 0.0006746 | 1.126 | 0.600 | 60 | 4.689 | 2.637 | 13.860 | L |
| 29 | 8.157 | 222.350 | 0.062 | 0.0007611 | 0.941 | 0.595 | 60 | 4.368 | 2.457 | 16.174 | L |
| 30 | 8.150 | 224.592 | 0.062 | 0.0007637 | 0.970 | 0.595 | 60 | 4.356 | 2.450 | 16.262 | L |
| 31 | 8.157 | 227.535 | 0.062 | 0.00074 | 0.974 | 0.599 | 60 | 4.390 | 2.470 | 15.792 | L |
| 32 | 8.150 | 228.063 | 0.062 | 0.0007379 | 0.975 | 0.600 | 60 | 4.393 | 2.471 | 15.758 | L |
| 33 | 6.541 | 238.653 | 0.053 | 0.0006565 | 0.779 | 0.595 | 60 | 4.402 | 2.476 | 15.951 | L |
| 34 | 6.510 | 239.622 | 0.053 | 0.0006558 | 0.775 | 0.595 | 60 | 4.400 | 2.475 | 15.981 | L |
| 35 | 6.510 | 239.622 | 0.053 | 0.0006558 | 0.775 | 0.595 | 60 | 4.400 | 2.475 | 15.981 | L |
| 36 | 6.510 | 242.278 | 0.052 | 0.0006558 | 0.878 | 0.595 | 60 | 4.393 | 2.471 | 16.002 | L |
| 37 | 5.467 | 247.724 | 0.046 | 0.00061 | 0.696 | 0.600 | 60 | 4.178 | 2.350 | 17.238 | L |
| 38 | 5.423 | 251.816 | 0.046 | 0.0006097 | 0.756 | 0.599 | 60 | 4.166 | 2.344 | 17.338 | L |
| 39 | 5.412 | 252.442 | 0.046 | 0.0006072 | 0.756 | 0.600 | 60 | 4.169 | 2.345 | 17.294 | L |
| 40 | 5.423 | 254.365 | 0.045 | 0.0005575 | 0.758 | 0.600 | 60 | 4.445 | 2.500 | 15.410 | L |
| 41 | 5.446 | 286.415 | 0.045 | 0.0005252 | 0.742 | 0.599 | 51 | 4.643 | 2.612 | 14.157 | L |
| 42 | 5.423 | 484.203 | 0.045 | 0.0005171 | 0.757 | 0.599 | 38 | 4.681 | 2.633 | 13.919 | L |

section are screened to reduce the number of feasible solution set. The application of various steps of *intermediate analysis* is explained in the following subsection.

7.1 Identification of new set of objectives

For decision making the following parameters of the motor are taken into consideration

Table 9 The results of screening for the motor with $N_s = 12$ $N_p = 8$

| Direction Importance Alternative | L M L_{stack} | L H Losses | L VH Mass | L H Time constant | L L J_{mot} | H H DiDo | L M N_{turns} | L VL wt | L L wy | H H Aslots | Final value |
|--|-----------------------|------------------|-----------------|-------------------------|---------------------|----------------|-----------------------|---------------|--------------|------------------|-------------|
| 1 | 39.153 | 122.200 | 0.247 | 0.0020986 | 1.856 | 0.506 | 60 | 1.727 | 1.295 | 45.748 | VL |
| 2 | 36.786 | 130.723 | 0.205 | 0.0019278 | 3.608 | 0.587 | 60 | 1.144 | 0.858 | 45.602 | VL |
| 3 | 36.793 | 131.379 | 0.201 | 0.0019101 | 3.816 | 0.599 | 59 | 1.050 | 0.788 | 45.489 | VL |
| 4 | 32.095 | 133.581 | 0.200 | 0.0019214 | 1.602 | 0.527 | 60 | 1.707 | 1.280 | 44.196 | VL |
| 5 | 33.028 | 135.156 | 0.190 | 0.0018583 | 3.640 | 0.583 | 60 | 1.249 | 0.937 | 44.705 | VL |
| 6 | 33.052 | 136.475 | 0.184 | 0.0017921 | 3.624 | 0.600 | 60 | 1.197 | 0.898 | 43.558 | L |
| 7 | 29.610 | 138.134 | 0.171 | 0.0015522 | 2.154 | 0.599 | 60 | 1.608 | 1.206 | 38.515 | L |
| 8 | 22.184 | 140.240 | 0.135 | 0.0015364 | 1.736 | 0.587 | 60 | 1.547 | 1.160 | 40.495 | L |
| 9 | 22.708 | 146.473 | 0.134 | 0.0014637 | 1.653 | 0.600 | 60 | 1.596 | 1.197 | 38.637 | L |
| 10 | 22.171 | 147.055 | 0.131 | 0.0014768 | 1.626 | 0.599 | 60 | 1.550 | 1.162 | 39.260 | L |
| 11 | 18.358 | 152.854 | 0.114 | 0.0012214 | 1.279 | 0.599 | 60 | 2.017 | 1.513 | 33.782 | M |
| 12 | 18.294 | 153.270 | 0.114 | 0.0012197 | 1.274 | 0.600 | 60 | 2.015 | 1.511 | 33.781 | M |
| 13 | 18.263 | 153.651 | 0.113 | 0.0012194 | 1.271 | 0.600 | 60 | 2.014 | 1.511 | 33.791 | M |
| 14 | 13.327 | 172.092 | 0.087 | 0.0011676 | 0.977 | 0.596 | 60 | 1.840 | 1.380 | 36.089 | M |
| 15 | 13.255 | 175.962 | 0.086 | 0.0011509 | 0.977 | 0.600 | 60 | 1.844 | 1.383 | 35.738 | M |
| 16 | 13.216 | 176.470 | 0.086 | 0.0011504 | 0.964 | 0.600 | 60 | 1.841 | 1.381 | 35.764 | M |
| 17 | 11.291 | 181.938 | 0.077 | 0.0010364 | 0.818 | 0.587 | 60 | 2.129 | 1.597 | 33.599 | M |
| 18 | 8.968 | 195.128 | 0.064 | 0.0005925 | 1.191 | 0.597 | 60 | 3.318 | 2.489 | 20.406 | L |
| 19 | 6.874 | 197.068 | 0.051 | 0.0005788 | 0.732 | 0.596 | 60 | 3.016 | 2.262 | 23.270 | L |
| 20 | 6.778 | 199.365 | 0.051 | 0.0005999 | 0.755 | 0.599 | 60 | 2.864 | 2.148 | 24.544 | L |
| 21 | 6.802 | 202.215 | 0.050 | 0.0005466 | 0.759 | 0.600 | 60 | 3.113 | 2.335 | 22.102 | L |
| 22 | 5.233 | 213.024 | 0.042 | 0.0004887 | 0.677 | 0.583 | 60 | 3.198 | 2.398 | 22.549 | L |

- Stack length,
- losses,
- mass,
- electrical time constant,
- inertia of the rotor,
- ratio of inner diameter of stator to outer diameter,
- number of turns,
- width of the tooth,
- thickness of the stator yoke,
- area of slots.

The losses and mass of the motor are the primary parameters. A motor with smallest losses and smallest mass is preferable. However as can be seen from the results of the previous section as the mass increases the losses decrease. Hence in the *intermediate analysis* both are considered for the screening purpose.

Electrical time constant of the motor has a direct influence on the dynamic performance of the motor. A motor with lower time constant has a better dynamic response compared to the motor with higher electrical time constant. Similarly the inertia of the rotor is important parameter because it influences the dynamic performance of the motor.

A motor with high inertia will accelerate slowly compared to the motor with lower inertia. The ratio of inner diameter of stator to outer diameter of stator is considered because it has an influence on the end turn of the winding.

The magnetic loading and the mechanical aspects determine the width of the tooth. If the tooth is too thin then it may not be able to withstand the mechanical forces acting on it. Hence in this analysis tooth with higher thickness is preferred.

The thickness required for the stator yoke depends on the magnetic loading of the machine as well as on the mechanical properties. If the number of the pole pairs is small, often the allowable magnetic loading and the mechanical loading determines the thickness of the stator yoke. However, if the number of pole pairs is high enough the stator yoke may be thin if it is sized according to the allowed magnetic loading. The mechanical constraints may thus determine the minimum thickness of the stator yoke. In the decision making process it smaller the thickness of stator yoke the better it is. A smaller yoke thickness is preferred because it reduces the mass of the steel lamination required.

Table 10 The results of screening for the motor with $N_s = 12$ $N_p = 10$

| Direction Importance Alternative | L M L _{stack} | L H Losses | L VH Mass | L H Time constant | L L J _{mot} | H H DiDo | L M N _{turns} | L VL wt | L L wy | H H Aslots | Final value |
|--|------------------------------|------------------|-----------------|-------------------------|----------------------------|----------------|------------------------------|---------------|--------------|------------------|-------------|
| 1 | 25.781 | 114.536 | 0.168 | 0.0023246 | 1.794 | 0.504 | 60 | 1.848 | 1.109 | 46.480 | VL |
| 2 | 25.862 | 118.012 | 0.168 | 0.0023281 | 1.868 | 0.509 | 60 | 1.780 | 1.068 | 46.935 | VL |
| 3 | 25.787 | 122.454 | 0.157 | 0.001965 | 2.541 | 0.567 | 60 | 1.799 | 1.080 | 41.582 | VL |
| 4 | 24.189 | 124.206 | 0.147 | 0.0020185 | 2.385 | 0.567 | 60 | 1.611 | 0.967 | 43.737 | VL |
| 5 | 20.531 | 130.141 | 0.139 | 0.002161 | 1.383 | 0.504 | 60 | 1.926 | 1.155 | 45.514 | VL |
| 6 | 20.488 | 131.790 | 0.138 | 0.0021564 | 1.383 | 0.504 | 60 | 1.929 | 1.157 | 45.447 | VL |
| 7 | 20.524 | 136.966 | 0.131 | 0.001778 | 2.113 | 0.564 | 60 | 2.013 | 1.208 | 39.406 | L |
| 8 | 17.490 | 137.605 | 0.123 | 0.0018707 | 1.179 | 0.504 | 60 | 2.370 | 1.422 | 39.953 | L |
| 9 | 17.490 | 138.022 | 0.122 | 0.0019735 | 1.179 | 0.504 | 60 | 2.128 | 1.277 | 42.941 | L |
| 10 | 17.493 | 143.015 | 0.120 | 0.0020916 | 1.188 | 0.505 | 59 | 1.844 | 1.106 | 46.425 | L |
| 11 | 18.237 | 145.270 | 0.115 | 0.0015907 | 1.994 | 0.589 | 60 | 2.000 | 1.200 | 37.330 | L |
| 12 | 18.185 | 146.264 | 0.114 | 0.0015779 | 2.006 | 0.591 | 60 | 2.002 | 1.201 | 37.123 | L |
| 13 | 17.386 | 149.075 | 0.111 | 0.0018453 | 1.852 | 0.567 | 60 | 1.610 | 0.966 | 43.768 | L |
| 14 | 14.858 | 150.121 | 0.106 | 0.0019816 | 1.077 | 0.504 | 60 | 1.868 | 1.121 | 46.229 | L |
| 15 | 14.895 | 152.044 | 0.104 | 0.0019974 | 1.159 | 0.510 | 60 | 1.764 | 1.059 | 47.074 | L |
| 16 | 14.044 | 154.792 | 0.103 | 0.001804 | 1.080 | 0.504 | 60 | 2.205 | 1.323 | 42.000 | L |
| 17 | 14.059 | 155.033 | 0.102 | 0.001901 | 1.058 | 0.504 | 60 | 1.970 | 1.182 | 44.958 | L |
| 18 | 14.059 | 156.596 | 0.100 | 0.001957 | 1.093 | 0.509 | 60 | 1.770 | 1.062 | 47.020 | L |
| 19 | 14.850 | 159.208 | 0.099 | 0.0015843 | 1.581 | 0.567 | 60 | 2.037 | 1.222 | 38.914 | L |
| 20 | 14.925 | 160.043 | 0.098 | 0.0017538 | 1.592 | 0.567 | 60 | 1.616 | 0.970 | 43.695 | L |
| 21 | 14.904 | 163.174 | 0.097 | 0.0016122 | 1.518 | 0.580 | 60 | 1.794 | 1.077 | 40.395 | L |
| 22 | 14.084 | 164.641 | 0.094 | 0.0016468 | 1.515 | 0.567 | 60 | 1.796 | 1.078 | 41.617 | L |
| 23 | 14.029 | 165.159 | 0.093 | 0.001715 | 1.509 | 0.567 | 60 | 1.619 | 0.972 | 43.655 | L |
| 24 | 14.075 | 166.999 | 0.093 | 0.0016861 | 1.551 | 0.572 | 60 | 1.630 | 0.978 | 43.035 | L |
| 25 | 14.062 | 167.051 | 0.092 | 0.0016858 | 1.537 | 0.572 | 60 | 1.630 | 0.978 | 43.042 | v |
| 26 | 14.029 | 170.280 | 0.090 | 0.0014998 | 1.717 | 0.600 | 60 | 1.731 | 1.039 | 39.113 | M |
| 27 | 10.332 | 176.067 | 0.072 | 0.0009414 | 1.109 | 0.600 | 60 | 2.967 | 1.780 | 26.366 | L |
| 28 | 10.383 | 178.156 | 0.071 | 0.0012403 | 1.115 | 0.600 | 60 | 2.003 | 1.202 | 36.250 | M |
| 29 | 10.332 | 179.231 | 0.071 | 0.0012376 | 1.109 | 0.600 | 60 | 2.003 | 1.202 | 36.250 | M |
| 30 | 6.585 | 183.990 | 0.050 | 0.0007325 | 0.771 | 0.600 | 60 | 3.021 | 1.813 | 25.854 | L |
| 31 | 6.585 | 184.051 | 0.050 | 0.000732 | 0.771 | 0.600 | 60 | 3.023 | 1.814 | 25.835 | L |
| 32 | 6.555 | 185.535 | 0.050 | 0.0007257 | 0.797 | 0.600 | 60 | 3.044 | 1.826 | 25.654 | L |
| 33 | 6.585 | 186.427 | 0.050 | 0.0007195 | 0.788 | 0.600 | 60 | 3.076 | 1.845 | 25.340 | L |
| 34 | 6.555 | 187.326 | 0.050 | 0.0007177 | 0.784 | 0.600 | 60 | 3.075 | 1.845 | 25.347 | L |
| 35 | 6.505 | 189.058 | 0.049 | 0.0007135 | 0.778 | 0.600 | 60 | 3.080 | 1.848 | 25.304 | L |
| 36 | 6.504 | 193.031 | 0.049 | 0.0006939 | 0.818 | 0.600 | 60 | 3.167 | 1.900 | 24.514 | L |
| 37 | 6.504 | 200.664 | 0.049 | 0.0006823 | 0.799 | 0.600 | 59 | 3.220 | 1.932 | 24.045 | L |
| 38 | 6.504 | 203.785 | 0.049 | 0.0006724 | 0.846 | 0.600 | 59 | 3.266 | 1.959 | 23.643 | L |
| 39 | 5.312 | 207.725 | 0.043 | 0.0006363 | 0.655 | 0.600 | 59 | 3.055 | 1.833 | 25.529 | L |
| 40 | 5.312 | 208.434 | 0.043 | 0.0006303 | 0.668 | 0.600 | 60 | 3.086 | 1.852 | 25.254 | L |
| 41 | 5.279 | 210.099 | 0.042 | 0.0006262 | 0.664 | 0.600 | 60 | 3.092 | 1.855 | 25.186 | L |
| 42 | 5.021 | 217.749 | 0.041 | 0.0006347 | 0.647 | 0.600 | 60 | 2.952 | 1.771 | 26.493 | L |
| 43 | 5.026 | 233.215 | 0.041 | 0.0005823 | 0.677 | 0.600 | 58 | 3.221 | 1.933 | 24.037 | L |

Table 11 Results of final screening process

| Direction Importance Alternative | L M L _{stack} | L H Losses | L VH Mass | L H Time constant | L L Jmot | H H DiDo | L M N _{turns} | L VL wt | L VL wy | H H Aslots | Ns | Np | Final value |
|--|------------------------------|------------------|-----------------|-------------------------|----------------|----------------|------------------------------|---------------|---------------|------------------|----|----|-------------|
| 1 | 6.583 | 381.327 | 0.054 | 0.0003302 | 0.879 | 0.581 | 47 | 5.765 | 4.324 | 5.908 | 6 | 4 | L |
| 2 | 6.551 | 396.263 | 0.052 | 0.0003356 | 0.875 | 0.600 | 47 | 5.414 | 4.060 | 6.642 | 6 | 4 | L |
| 3 | 5.863 | 457.472 | 0.049 | 0.0003225 | 0.708 | 0.581 | 44 | 5.645 | 4.234 | 6.355 | 6 | 4 | L |
| 4 | 8.427 | 196.408 | 0.063 | 0.0004116 | 1.381 | 0.600 | 60 | 4.680 | 3.510 | 10.254 | 9 | 6 | L |
| 5 | 7.344 | 197.133 | 0.055 | 0.0004021 | 1.033 | 0.600 | 60 | 4.548 | 3.411 | 11.035 | 9 | 6 | L |
| 6 | 7.331 | 198.013 | 0.055 | 0.0004066 | 0.995 | 0.600 | 60 | 4.520 | 3.390 | 11.204 | 9 | 6 | L |
| 7 | 6.667 | 211.152 | 0.052 | 0.0003902 | 1.150 | 0.600 | 60 | 4.478 | 3.358 | 11.467 | 9 | 6 | L |
| 8 | 6.641 | 213.407 | 0.052 | 0.0004011 | 1.052 | 0.600 | 60 | 4.409 | 3.307 | 11.897 | 9 | 6 | L |
| 9 | 6.650 | 215.335 | 0.052 | 0.0004138 | 0.965 | 0.600 | 60 | 4.340 | 3.255 | 12.347 | 9 | 6 | L |
| 10 | 6.642 | 218.685 | 0.051 | 0.0004337 | 0.849 | 0.600 | 60 | 4.228 | 3.171 | 13.092 | 9 | 6 | L |
| 11 | 6.641 | 218.706 | 0.051 | 0.0004337 | 0.848 | 0.600 | 60 | 4.228 | 3.171 | 13.095 | 9 | 6 | L |
| 12 | 6.659 | 241.040 | 0.051 | 0.0003775 | 0.936 | 0.600 | 52 | 4.550 | 3.413 | 11.021 | 9 | 6 | L |
| 13 | 5.231 | 253.763 | 0.044 | 0.0003926 | 0.933 | 0.600 | 58 | 4.085 | 3.063 | 14.091 | 9 | 6 | L |
| 14 | 5.231 | 254.492 | 0.044 | 0.0003927 | 0.932 | 0.600 | 58 | 4.084 | 3.063 | 14.095 | 9 | 6 | L |
| 15 | 5.146 | 257.163 | 0.043 | 0.0003925 | 0.917 | 0.600 | 58 | 4.058 | 3.043 | 14.282 | 9 | 6 | L |
| 16 | 5.176 | 261.857 | 0.043 | 0.0003957 | 0.867 | 0.600 | 60 | 4.048 | 3.036 | 14.353 | 9 | 6 | L |
| 17 | 5.191 | 264.120 | 0.043 | 0.0004153 | 0.753 | 0.600 | 60 | 3.934 | 2.950 | 15.191 | 9 | 6 | L |
| 18 | 5.191 | 266.084 | 0.043 | 0.0004225 | 0.705 | 0.600 | 60 | 3.891 | 2.918 | 15.513 | 9 | 6 | L |
| 19 | 5.155 | 267.243 | 0.043 | 0.0004205 | 0.696 | 0.600 | 60 | 3.891 | 2.918 | 15.513 | 9 | 6 | L |
| 20 | 5.137 | 269.628 | 0.043 | 0.0004277 | 0.678 | 0.600 | 60 | 3.842 | 2.882 | 15.885 | 9 | 6 | L |
| 21 | 5.146 | 307.658 | 0.042 | 0.0003999 | 0.699 | 0.600 | 52 | 4.013 | 3.010 | 14.609 | 9 | 6 | L |
| 22 | 5.138 | 362.249 | 0.042 | 0.0003071 | 0.806 | 0.600 | 42 | 4.619 | 3.464 | 10.609 | 9 | 6 | L |
| 23 | 5.150 | 370.205 | 0.041 | 0.000323 | 0.699 | 0.600 | 42 | 4.512 | 3.384 | 11.253 | 9 | 6 | L |
| 24 | 5.150 | 370.205 | 0.041 | 0.000323 | 0.699 | 0.600 | 42 | 4.512 | 3.384 | 11.253 | 9 | 6 | L |
| 25 | 5.144 | 439.331 | 0.041 | 0.0002639 | 0.780 | 0.600 | 34 | 4.939 | 3.704 | 8.840 | 9 | 6 | L |
| 26 | 26.491 | 142.312 | 0.182 | 0.0028708 | 1.840 | 0.501 | 60 | 2.347 | 1.320 | 41.122 | 9 | 8 | VL |
| 27 | 26.574 | 143.091 | 0.182 | 0.0028883 | 1.740 | 0.501 | 59 | 2.318 | 1.304 | 41.472 | 9 | 8 | VL |
| 28 | 25.288 | 146.151 | 0.173 | 0.0029022 | 1.658 | 0.501 | 59 | 2.212 | 1.244 | 42.744 | 9 | 8 | VL |
| 29 | 24.557 | 150.596 | 0.165 | 0.0028077 | 2.036 | 0.521 | 59 | 2.062 | 1.160 | 43.054 | 9 | 8 | VL |
| 30 | 24.673 | 151.189 | 0.164 | 0.0028218 | 2.134 | 0.521 | 60 | 2.039 | 1.147 | 43.321 | 9 | 8 | VL |
| 31 | 24.635 | 153.936 | 0.162 | 0.0026879 | 2.286 | 0.541 | 60 | 2.006 | 1.129 | 42.070 | 9 | 8 | VL |
| 31 | 26.522 | 158.045 | 0.162 | 0.0023082 | 2.768 | 0.595 | 60 | 2.061 | 1.159 | 36.646 | 9 | 8 | VL |
| 33 | 22.448 | 159.025 | 0.152 | 0.0027394 | 1.963 | 0.525 | 59 | 1.997 | 1.123 | 43.552 | 9 | 8 | VL |
| 34 | 22.440 | 161.063 | 0.150 | 0.0026146 | 2.082 | 0.541 | 60 | 2.005 | 1.128 | 42.091 | 9 | 8 | VL |
| 35 | 19.888 | 170.183 | 0.138 | 0.0026272 | 1.821 | 0.521 | 60 | 2.063 | 1.160 | 43.030 | 9 | 8 | VL |
| 36 | 19.772 | 174.089 | 0.136 | 0.0024361 | 1.914 | 0.542 | 60 | 2.154 | 1.212 | 40.312 | 9 | 8 | VL |
| 37 | 19.874 | 175.847 | 0.133 | 0.0023942 | 1.837 | 0.553 | 60 | 2.090 | 1.176 | 40.139 | 9 | 8 | VL |
| 38 | 19.874 | 178.268 | 0.133 | 0.0024049 | 1.836 | 0.553 | 60 | 2.068 | 1.163 | 40.394 | 9 | 8 | VL |
| 39 | 17.181 | 182.271 | 0.117 | 0.0015246 | 1.579 | 0.595 | 60 | 3.204 | 1.802 | 25.286 | 9 | 8 | M |
| 40 | 17.283 | 183.240 | 0.117 | 0.0015224 | 1.558 | 0.595 | 60 | 3.219 | 1.811 | 25.153 | 9 | 8 | M |
| 41 | 17.252 | 184.765 | 0.115 | 0.0015063 | 1.450 | 0.600 | 60 | 3.191 | 1.795 | 25.085 | 9 | 8 | M |
| 42 | 17.216 | 185.155 | 0.115 | 0.0015052 | 1.448 | 0.600 | 60 | 3.190 | 1.794 | 25.089 | 9 | 8 | M |
| 43 | 16.087 | 192.939 | 0.110 | 0.0015347 | 1.391 | 0.597 | 59 | 3.048 | 1.715 | 26.571 | 9 | 8 | M |
| 44 | 8.241 | 198.313 | 0.063 | 0.000679 | 1.129 | 0.599 | 60 | 4.679 | 2.632 | 13.932 | 9 | 8 | L |
| 45 | 8.213 | 199.209 | 0.062 | 0.0006746 | 1.126 | 0.600 | 60 | 4.689 | 2.637 | 13.860 | 9 | 8 | L |
| 46 | 8.157 | 222.350 | 0.062 | 0.0007611 | 0.941 | 0.595 | 60 | 4.368 | 2.457 | 16.174 | 9 | 8 | L |
| 47 | 8.150 | 224.592 | 0.062 | 0.0007637 | 0.970 | 0.595 | 60 | 4.356 | 2.450 | 16.262 | 9 | 8 | L |

Table 11 continued

| Direction Importance Alternative | L M L _{stack} | L H Losses | L VH Mass | L H Time constant | L L Jmot | H H DiDo | L M N _{turns} | L VL wt | L VL wy | H H Aslots | Ns | Np | Final value |
|--|------------------------------|------------------|-----------------|-------------------------|----------------|----------------|------------------------------|---------------|---------------|------------------|----|----|-------------|
| 48 | 8.157 | 227.535 | 0.062 | 0.00074 | 0.974 | 0.599 | 60 | 4.390 | 2.470 | 15.792 | 9 | 8 | L |
| 49 | 8.150 | 228.063 | 0.062 | 0.0007379 | 0.975 | 0.600 | 60 | 4.393 | 2.471 | 15.758 | 9 | 8 | L |
| 50 | 6.541 | 238.653 | 0.053 | 0.0006565 | 0.779 | 0.595 | 60 | 4.402 | 2.476 | 15.951 | 9 | 8 | L |
| 51 | 6.510 | 239.622 | 0.053 | 0.0006558 | 0.775 | 0.595 | 60 | 4.400 | 2.475 | 15.981 | 9 | 8 | L |
| 52 | 6.510 | 239.622 | 0.053 | 0.0006558 | 0.775 | 0.595 | 60 | 4.400 | 2.475 | 15.981 | 9 | 8 | L |
| 53 | 6.510 | 242.278 | 0.052 | 0.0006558 | 0.878 | 0.595 | 60 | 4.393 | 2.471 | 16.002 | 9 | 8 | L |
| 54 | 5.467 | 247.724 | 0.046 | 0.00061 | 0.696 | 0.600 | 60 | 4.178 | 2.350 | 17.238 | 9 | 8 | L |
| 55 | 5.423 | 251.816 | 0.046 | 0.0006097 | 0.756 | 0.599 | 60 | 4.166 | 2.344 | 17.338 | 9 | 8 | L |
| 56 | 5.412 | 252.442 | 0.046 | 0.0006072 | 0.756 | 0.600 | 60 | 4.169 | 2.345 | 17.294 | 9 | 8 | L |
| 57 | 5.423 | 254.365 | 0.045 | 0.0005575 | 0.758 | 0.600 | 60 | 4.445 | 2.500 | 15.410 | 9 | 8 | L |
| 58 | 5.446 | 286.415 | 0.045 | 0.0005252 | 0.742 | 0.599 | 51 | 4.643 | 2.612 | 14.157 | 9 | 8 | L |
| 59 | 5.423 | 484.203 | 0.045 | 0.0005171 | 0.757 | 0.599 | 38 | 4.681 | 2.633 | 13.919 | 9 | 8 | L |
| 60 | 20.866 | 136.246 | 0.147 | 0.0025642 | 1.811 | 0.516 | 60 | 2.642 | 1.189 | 38.770 | 9 | 10 | VL |
| 61 | 18.946 | 143.236 | 0.138 | 0.0024483 | 1.492 | 0.508 | 60 | 2.841 | 1.279 | 37.140 | 9 | 10 | VL |
| 62 | 17.614 | 149.605 | 0.130 | 0.002309 | 1.378 | 0.508 | 60 | 3.018 | 1.358 | 35.261 | 9 | 10 | VL |
| 63 | 15.732 | 159.273 | 0.118 | 0.0023229 | 1.240 | 0.508 | 60 | 2.738 | 1.232 | 38.254 | 9 | 10 | VL |
| 64 | 14.713 | 166.044 | 0.111 | 0.0022571 | 1.162 | 0.508 | 60 | 2.740 | 1.233 | 38.202 | 9 | 10 | VL |
| 65 | 13.751 | 175.540 | 0.106 | 0.0021383 | 1.072 | 0.508 | 60 | 2.882 | 1.297 | 36.682 | 9 | 10 | VL |
| 66 | 13.788 | 180.009 | 0.103 | 0.0020036 | 1.270 | 0.537 | 60 | 2.788 | 1.254 | 35.755 | 9 | 10 | VL |
| 67 | 8.046 | 205.460 | 0.062 | 0.0008258 | 1.014 | 0.600 | 60 | 4.478 | 2.015 | 17.609 | 9 | 10 | VL |
| 68 | 8.009 | 207.324 | 0.062 | 0.0008158 | 1.010 | 0.600 | 60 | 4.514 | 2.031 | 17.371 | 9 | 10 | VL |
| 69 | 5.077 | 254.040 | 0.046 | 0.0006926 | 0.651 | 0.599 | 60 | 3.993 | 1.797 | 21.016 | 9 | 10 | VL |
| 70 | 18.358 | 152.854 | 0.114 | 0.0012214 | 1.279 | 0.599 | 60 | 2.017 | 1.513 | 33.782 | 12 | 8 | M |
| 71 | 18.294 | 153.270 | 0.114 | 0.0012197 | 1.274 | 0.600 | 60 | 2.015 | 1.511 | 33.781 | 12 | 8 | M |
| 72 | 18.263 | 153.651 | 0.113 | 0.0012194 | 1.271 | 0.600 | 60 | 2.014 | 1.511 | 33.791 | 12 | 8 | M |
| 73 | 13.327 | 172.092 | 0.087 | 0.0011676 | 0.977 | 0.596 | 60 | 1.840 | 1.380 | 36.089 | 12 | 8 | M |
| 74 | 13.255 | 175.962 | 0.086 | 0.0011509 | 0.977 | 0.600 | 60 | 1.844 | 1.383 | 35.738 | 12 | 8 | M |
| 75 | 13.216 | 176.470 | 0.086 | 0.0011504 | 0.964 | 0.600 | 60 | 1.841 | 1.381 | 35.764 | 12 | 8 | M |
| 76 | 11.291 | 181.938 | 0.077 | 0.0010364 | 0.818 | 0.587 | 60 | 2.129 | 1.597 | 33.599 | 12 | 8 | M |
| 77 | 10.383 | 178.156 | 0.071 | 0.0012403 | 1.115 | 0.600 | 60 | 2.003 | 1.202 | 36.250 | 12 | 10 | M |
| 78 | 10.332 | 179.231 | 0.071 | 0.0012376 | 1.109 | 0.600 | 60 | 2.003 | 1.202 | 36.250 | 12 | 10 | M |
| 79 | 14.029 | 170.280 | 0.090 | 0.0014998 | 1.717 | 0.600 | 60 | 1.731 | 1.039 | 39.113 | 12 | 10 | M |

The area of the slot is considered as an objective because it influences the winding. A slot with smaller area is difficult to wind. Hence in this analysis a larger slot area is preferred.

7.2 Linguistic term set

For the screening purpose the Linguistic term set Based on the Ordered Structure is used. A set of seven terms of ordered structured linguistic terms is used here:

$$S = \{S_0 = \text{none}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{medium}, s_4 = \text{high}, s_5 = \text{very high}, s_6 = \text{perfect}\}$$

where $s_a < s_b$ iff $a < b$. The linguistic terms set in addition satisfy the following conditions:

- Negation operator: $Neg(s_i) = s_j, j = T - i$ ($T + 1$ is the cardinality),
- maximisation operator: $Max(s_i, s_j) = s_i$, if $s_i \geq s_j$,
- minimisation operator: $Min(s_i, s_j) = s_i$, if $s_i \leq s_j$.

7.3 The semantic of linguistic term set

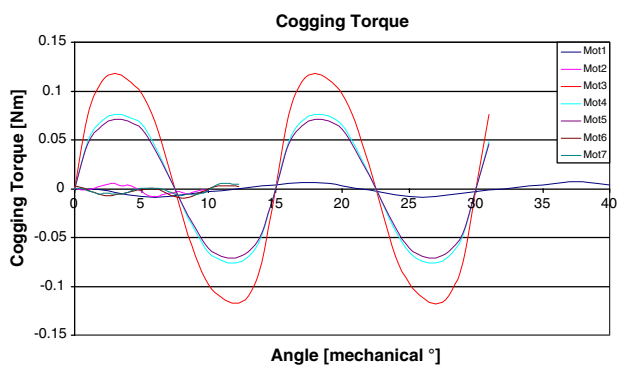
In this case the Semantic Based on the Ordered Structure is used. The terms are symmetrically distributed, i.e. it is

Table 12 Parameters of the set of solutions

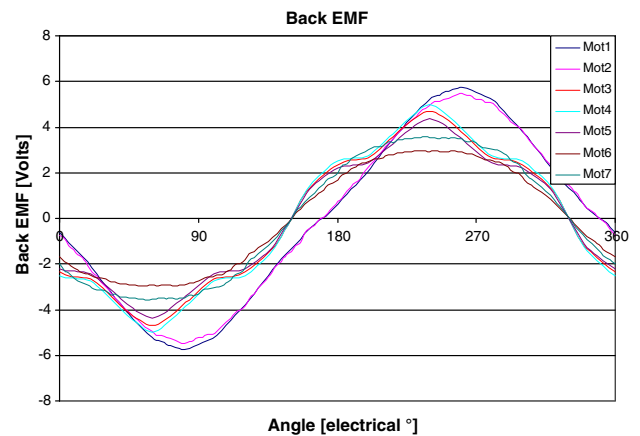
| Alternative number | L_{stack} | Losses | Mass | DiDo | α_m | hm | N_{turns} | Bmag | Biron | wt | wy | Ns | Np |
|--------------------|-------------|---------|-------|-------|------------|-------|-------------|------|-------|-------|-------|----|----|
| 1 | 17.181 | 182.271 | 0.117 | 0.595 | 0.587 | 1.784 | 60 | 1.19 | 1.51 | 3.204 | 1.802 | 9 | 8 |
| 2 | 17.283 | 183.240 | 0.117 | 0.595 | 0.616 | 1.675 | 60 | 1.16 | 1.51 | 3.219 | 1.811 | 9 | 8 |
| 3 | 17.252 | 184.765 | 0.115 | 0.600 | 0.601 | 1.574 | 60 | 1.18 | 1.51 | 3.191 | 1.795 | 9 | 8 |
| 4 | 17.216 | 185.155 | 0.115 | 0.600 | 0.601 | 1.575 | 60 | 1.18 | 1.51 | 3.190 | 1.794 | 9 | 8 |
| 5 | 16.087 | 192.939 | 0.110 | 0.597 | 0.587 | 1.674 | 59 | 1.20 | 1.58 | 3.048 | 1.715 | 9 | 8 |
| 6 | 18.358 | 152.854 | 0.114 | 0.599 | 0.523 | 1.505 | 60 | 1.20 | 1.58 | 2.017 | 1.513 | 12 | 8 |
| 7 | 18.294 | 153.270 | 0.114 | 0.600 | 0.522 | 1.505 | 60 | 1.20 | 1.58 | 2.015 | 1.511 | 12 | 8 |
| 8 | 18.263 | 153.651 | 0.113 | 0.600 | 0.522 | 1.505 | 60 | 1.20 | 1.58 | 2.014 | 1.511 | 12 | 8 |
| 9 | 13.327 | 172.092 | 0.087 | 0.596 | 0.547 | 1.534 | 60 | 1.12 | 1.68 | 1.840 | 1.380 | 12 | 8 |
| 10 | 13.255 | 175.962 | 0.086 | 0.600 | 0.546 | 1.522 | 60 | 1.12 | 1.68 | 1.844 | 1.383 | 12 | 8 |
| 11 | 13.216 | 176.470 | 0.086 | 0.600 | 0.547 | 1.504 | 60 | 1.12 | 1.68 | 1.841 | 1.381 | 12 | 8 |
| 12 | 11.291 | 181.938 | 0.077 | 0.587 | 0.577 | 1.505 | 60 | 1.18 | 1.58 | 2.129 | 1.597 | 12 | 8 |
| 13 | 10.383 | 178.156 | 0.071 | 0.600 | 0.802 | 1.510 | 60 | 0.81 | 1.53 | 2.003 | 1.202 | 12 | 10 |
| 14 | 10.332 | 179.231 | 0.071 | 0.600 | 0.802 | 1.510 | 60 | 0.81 | 1.53 | 2.003 | 1.202 | 12 | 10 |
| 15 | 14.029 | 170.280 | 0.090 | 0.600 | 0.838 | 1.636 | 60 | 0.82 | 1.91 | 1.731 | 1.039 | 12 | 10 |

Table 13 Parameters of the final set of solution

| Alternative number | L_{stack} | DiDo | α_m | hm | N_{turns} | Bmag | Biron | wt | wy | Ns | Np |
|--------------------|-------------|-------|------------|-------|-------------|-------|-------|-------|-------|----|----|
| 1 | 17.233 | 0.597 | 0.601 | 1.652 | 60 | 1.179 | 1.506 | 3.201 | 1.801 | 9 | 8 |
| 2 | 16.087 | 0.597 | 0.587 | 1.674 | 59 | 1.198 | 1.577 | 3.048 | 1.715 | 9 | 8 |
| 3 | 18.305 | 0.600 | 0.522 | 1.505 | 60 | 1.200 | 1.582 | 2.015 | 1.511 | 12 | 8 |
| 4 | 13.266 | 0.599 | 0.547 | 1.520 | 60 | 1.121 | 1.684 | 1.842 | 1.381 | 12 | 8 |
| 5 | 11.291 | 0.587 | 0.577 | 1.505 | 60 | 1.184 | 1.579 | 2.129 | 1.597 | 12 | 8 |
| 6 | 10.358 | 0.600 | 0.802 | 1.510 | 60 | 0.805 | 1.534 | 2.003 | 1.202 | 12 | 10 |
| 7 | 14.029 | 0.600 | 0.838 | 1.636 | 60 | 0.821 | 1.909 | 1.731 | 1.039 | 12 | 10 |

**Fig. 33** Cogging Torque for all the seven motors

assumed that linguistic term sets are distributed on a scale with an odd cardinal and the mid term representing an assessment of “approximately 0.5” and the rest of the terms are placed symmetrically around it.

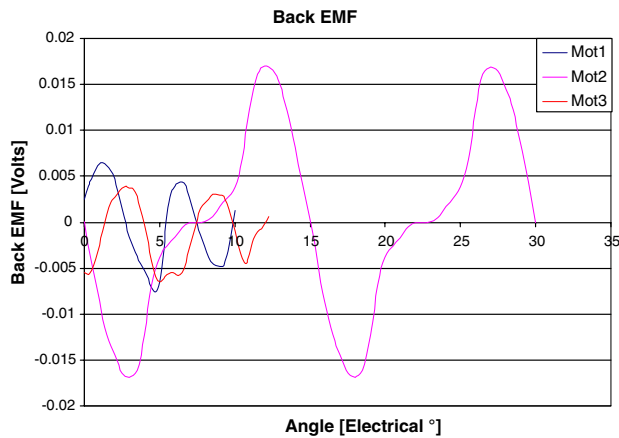
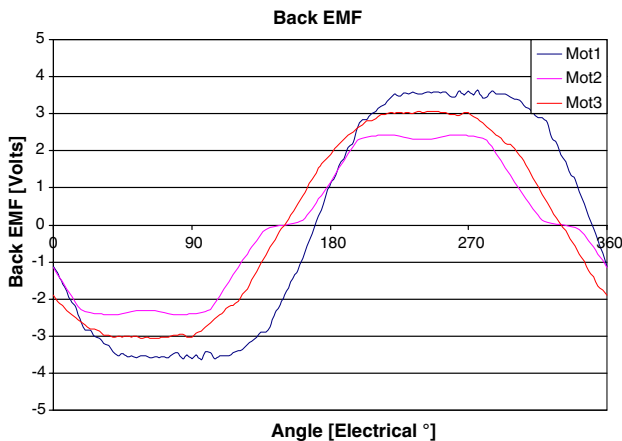
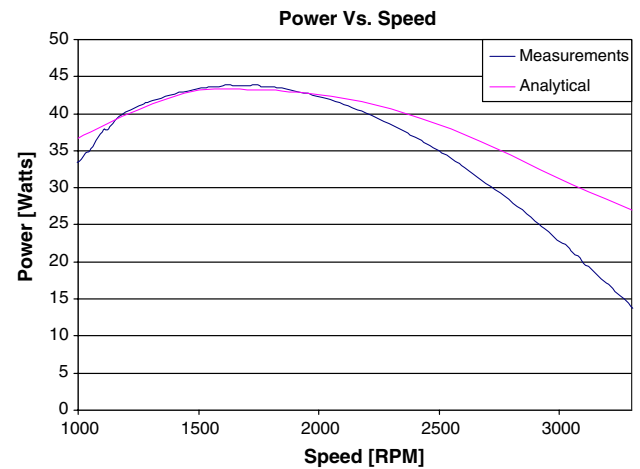
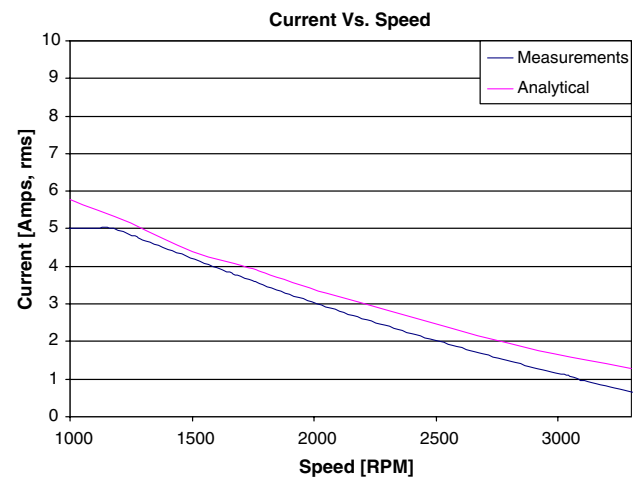
**Fig. 34** Back EMF for all the seven motors

7.4 Aggregation operator for linguistic weighted information

In this case the Linguistic Weighted conjunction aggregation operator is used.

Table 14 Parameters of the refine set of solution

| Alternative number | L_{stack} | DiDo | α_m | hm | N_{turns} | Bmag | Biron | wt | wy | Ns | Np |
|--------------------|--------------------|-------|------------|-------|--------------------|-------|-------|-------|-------|----|----|
| 1 | 10 | 0.597 | 1 | 1.652 | 60 | 1.179 | 1.506 | 3.201 | 1.801 | 9 | 8 |
| 2 | 10 | 0.6 | 1 | 1.505 | 60 | 0.65 | 1.577 | 2.015 | 1.511 | 12 | 8 |
| 3 | 18 | 0.5 | 1 | 1.636 | 60 | 0.82 | 1.909 | 1.731 | 1.039 | 12 | 10 |

**Fig. 35** Cogging Torque for the final three motors**Fig. 36** Back EMF for the final three motors**Fig. 37** Power versus speed characteristics comparison between simulation and experiments**Fig. 38** Current versus speed characteristic comparison between simulations and experiment values

7.5 The screening process

The importance of different parameters discussed in the previous section is shown in Table 4.

The length of the motor stack is given medium importance and the smaller the length of the motor the better it is, i.e. a smaller stack length is preferred over the larger length. For the losses a high importance is given and lower the losses the better. Similarly for the mass a very high importance is given and smaller the mass the more preferred is the motor. The electrical time constant of the motor is given a high importance and the lower value is

better. Ratio of inner to outer stator diameter is given a higher value and higher the value the better it is. A medium importance is given to number of turns and lower the number of turns is preferred. The remanence field of permanent magnet and maximum allowable field density of stator lamination is given no importance. The width of the tooth and width of the yoke are given very low and low importance, respectively and lower the values of both the parameters the better it is. The area of the slot is given a

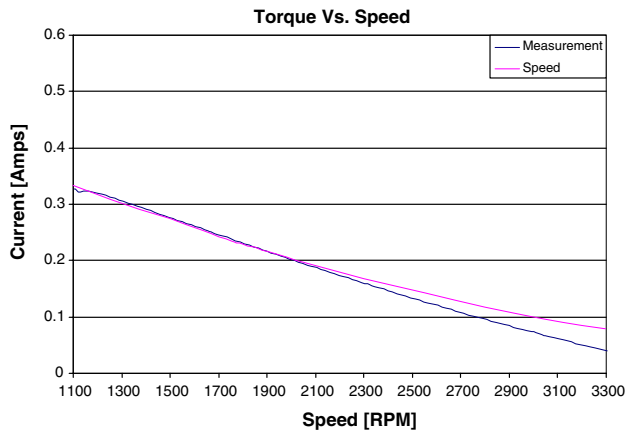


Fig. 39 Torque versus speed characteristic comparison

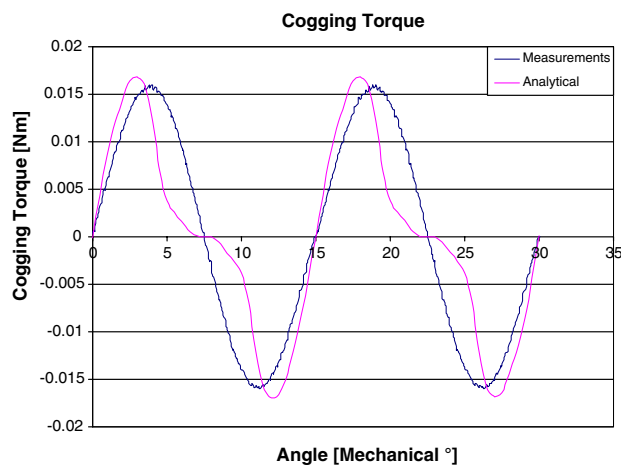


Fig. 40 Cogging torque comparison between simulation and experimental values

high importance and the higher value of the slot area is preferred. The results of the multicriteria decision for motors with different number of poles and slots are given in Tables 5, 6, 7, 8, 9, 10.

In the above table the motors selected for further analysis are marked in bold. The designs three to seven are equally good and are considered for further analysis. Hence in total the feasible solutions is 79 out of 195 total solutions. The first phase of screening process is able to eliminate 60% of solutions and only 40% of solutions are selected.

In the second phase of screening the 79 competent solutions obtained so far are again subjected to multicriteria decision making. The results of second phase of screening are given in Table 11.

From the second phase of screening 15 feasible solutions are obtained. All the solutions having the highest final value are chosen. The relevant parameters of the 15 solutions are given in Table 12. For all the motors the outer diameter of the motor is 40 mm and the air-gap length is 0.2 mm.

From the above table it can be seen that alternatives one to four have almost the same geometric parameters. Similarly 6 to 8 have similar parameters and 9 to 11 have almost similar parameters. The alternatives 13 and 14 are also similar geometrically. Hence in order to reduce the number of alternatives for detailed simulation the geometrically similar alternatives are replaced by single alternative and its geometric parameters are average of the similar alternatives. Hence instead of considering alternatives one to four, a single alternative with parameters that are average of parameters of alternatives one to four is considered. The new set of alternatives that are considered for detailed modelling are given in Table 13.

Hence these seven solutions that will be considered for detailed analysis. For detailed analysis FEM models of the motor are developed using FEMAG and smartFEM. In the section the results of final analysis are give.

8 Final analysis phase of progressive design methodology for design of a BLDC motor drive

In this section detailed analysis of the motors obtained in the previous section is done. For the detailed analysis FEM model of the motors were developed in FEMAG and smartFEM. The results of cogging torque and back emf for all the seven alternatives in Table 13 are shown in Figs. 33 and 34, respectively.

From the above figures it can be seen that motors three, four and five have higher cogging torque compared to other motors. None of the motors have trapezoidal back emf. To obtain trapezoidal back emf the geometric parameters of the motors have to be varied. The parameter that will have an impact on the shape of the back emf is *ratio of magnet angle to pole pitch* (α_m). The parameters that was varied is $Bmag$. The new dimensions of the motors are given in Table 14.

The values of back emf and cogging torques for the above motor configurations are shown in Figs. 35 and 36, respectively.

From Fig. 35 it is seen that all the three motors have a cogging torque of less than 20 mNm and the back emf of all the motors is as close to trapezoidal shape as possible. Hence all the three motors are competent. Finally a prototype based on configuration 2 given in Table 14 was made. The characteristics curves of the prototype are given in Figs. 37, 38, 39, 40.

The comparison between the simulated and experimental cogging torque is shown in Fig. 40. From the above figures it can be seen that the performance of the motor is close to the simulated values.

9 Conclusion

In this work the progressive design methodology (PDM) is proposed. This methodology is suitable for designing complex systems, such as electrical drive and power electronics, from conceptual stage to final design. The main aspects of PDM discussed are as follows:

- PDM allows effective and efficient practices and techniques to be used from the start of the project.
- PDM ensures that each component of the system is compatible with each other.
- The computation time required for optimisation is reduced as the bulk of optimisation is done in the synthesis phase and the models of the components of the target system are simple in the synthesis phase.
- The experience of design engineers and production engineers are included in the intermediate analysis thus ensuring that the target system is feasible to manufacture.

In PDM the decision making factor is critical as proper decisions about dimensions, features, materials, and performance in the conceptual stage will ensure a robust and optimal design of the system. The different stages of PDM are explained using the example of the design of a BLDC motor and the results are validated by experiments. It is shown that using PDM an optimal design of the motor can be obtained that meets the performance requirements.

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